CORRELATION FUNCTIONS USING LAGUERRE TYPE CIRCUITS

Jorge Pedro Auge

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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by

Jorge Pedro Auge

Thesis Advisor: O. Baycura

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Using
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by

Jorge Pedro Augé
Lieutenant, Argentine Navy
Ingeniero en Telecomunicaciones
Universidad de Buenos Aires
Argentina, 1961

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ABSTRACT

This work deals with the correlation functions and their methods of obtention. A method without a "pure" time delay is considered which uses Laguerre functions type filters. The possibilities of utilization of the nonsymmetric Laguerre type filter are analyzed. A very inexpensive set of circuits which are useful to obtain the mentioned filters for low frequencies signals are presented, together with their computer analysis and laboratory realizations.

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ACKNOWLEDGMENT

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To my wife and children, for their understanding and patience, I am deeply grateful.

I. INTRODUCTION

Correlation analysis has become the most powerful tool to separate the signal from the noise, especially in those applications where the signals are hidden beneath a blanket of other signals and extraneous noise.

The development of instruments which can continuously sample even the most noisy signals and compute the correlation function simultaneously, have extended the use of correlation analysis into radio astronomy, fluid and solid state physics, neurology, seismology, geophysical exploration and other areas [Ref. 3].

Radar and sonar, of primary importance in military applications, use correlation detectors because it has been shown, for the case of the binary decision problem, that no matter what the philosophy employed for the design of a receiver is, the best detector is always a crosscorrelator detector [Ref. 5].

Theoretically, an infinite signal to noise ratio can be obtained in the detection of periodic signals in noise through the correlation technique, but this would require an infinite time of observation. In practice, by using crosscorrelation practical measurements can be made in reasonable time.

Initially, theoretical considerations on correlation functions, their properties and methods to obtain them using

pure time delays will be presented. Next, this thesis will present a method which does not use a pure time delay and yields results comparable to classical methods.

The mathematical formulation of the method and the circuits that can be used are studied and compared with the standard methods. Finally an approximation to the necessary type of circuits was analyzed by digital computer methods and checked in the laboratory using actual components.

II. CORRELATION FUNCTIONS

It is assumed that some physical process produces the time functions $x_1(t)$, $x_2(t)$, ... $x_N(t)$ simultaneously. It is assumed further that the physical process is stationary, in other words it is not changing over time. Also it is assumed that the time functions are not zero and do not have a d.c. component or they have zero mean if the time functions correspond to a stochastic process [Ref. 3].

A. AUTOCORRELATION FUNCTION

The time average autocorrelation function for deterministic or known signals is defined for two different classes of signals, signals of finite average power (case II) and signals of finite energy or pulse signals (caseI).

For signals of case II the definition is:

$$\mathcal{R}(\tau) \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_1(t)\, x_1(t+\tau)\, dt \tag{1}$$

and for signals of case I:

$$\mathcal{R}(\tau) \triangleq \int_{-\infty}^{\infty} x_1(t)\, x_1(t+\tau)\, dt \tag{2}$$

If the time function under consideration is a stochastic process the autocorrelation function is defined as follows:

$$R(\tau) = E[x_1(t)\, x_1(t+\tau)] \tag{3}$$

If the stochastic process is ergodic

$$\mathcal{R}(\tau) = R(\tau) \tag{4}$$

According to the definitions given above, Figure 1 shows an elementary device to obtain the autocorrelation function. The process implies multiplication of the signal by its delayed replica and averaging. If the instantaneous values of $x_1(t)$ and $x_1(t + \tau)$ are multiplied and the product averaged over a sufficiently long time, the result is the autocorrelation function if some means to vary τ have been added to the circuit.

1. Properties of the Autocorrelation [Ref. 3]

The autocorrelation function has the following properties:

a. Maximum.

The autocorrelation function is a maximum at $\tau = 0$, mathematically:

$$|\mathcal{R}(\tau)| \leq \mathcal{R}(0) \qquad \text{Schwartz Inequality} \tag{5}$$

b. Value at $\tau = 0$

The value of the autocorrelation at $\tau = 0$ is connected to the total power of the signal. If $x_1(t)$ is a voltage, then for $\tau = 0$, $\overline{x_1(t)^2}$ is the average power of the signal (as measured on a 1 ohm resistor). Hence $\mathcal{R}(0)$ is the average power of signals of case II and $\mathcal{R}(0)$ is the energy in the pulse signals of case I.

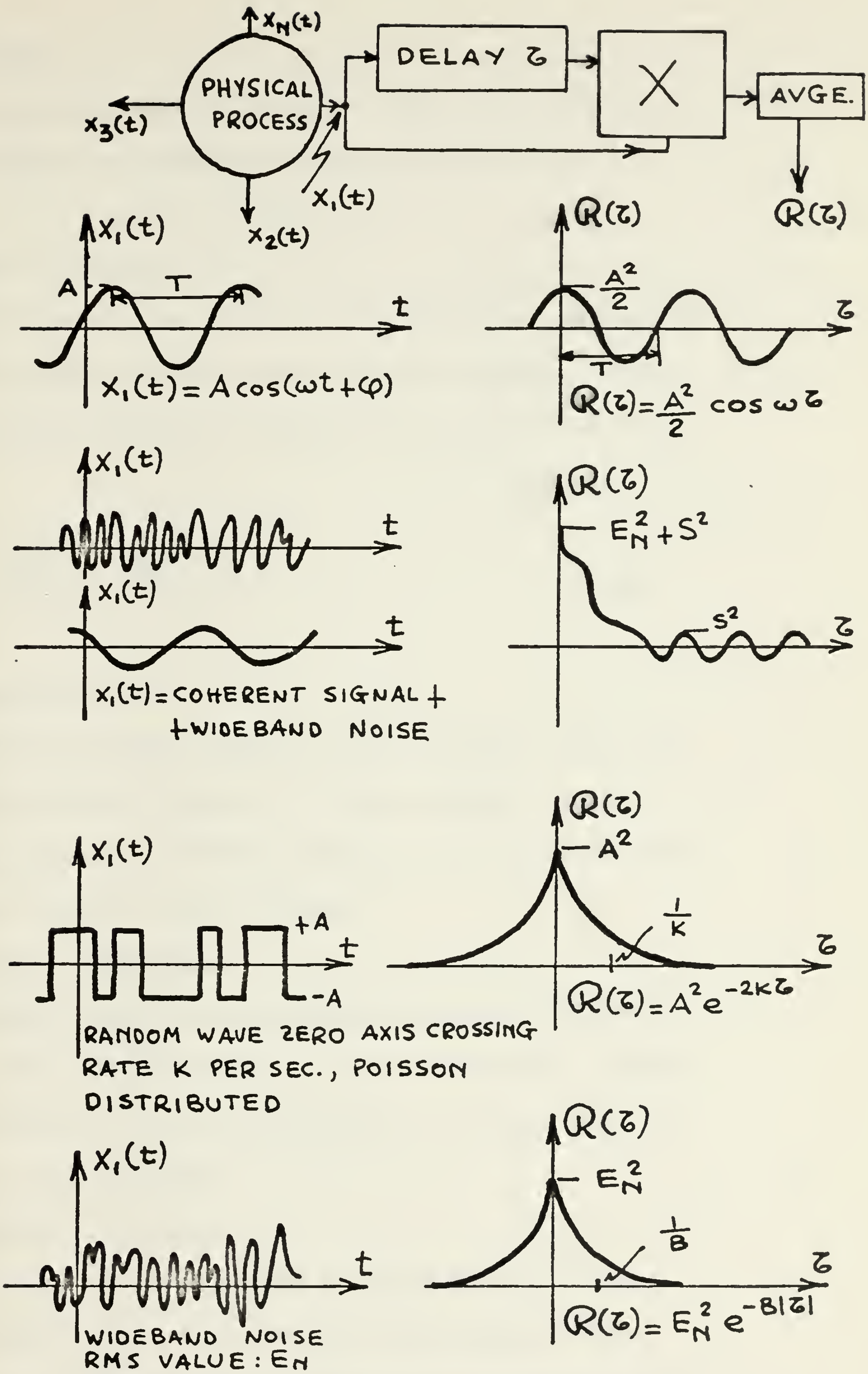


FIGURE 1: Typical Correlation Functions.

c. Function

The autocorrelation function is a function of the form of which is characteristic of the original gnal $x_1(t)$.

d. Type of Function

The value of the function for negative values of is identical to that for the same positive values. This ans that the autocorrelation function is an even function τ:

$$\mathcal{R}(\tau) = \mathcal{R}(-\tau) \tag{6}$$

e. Averaging Time

If the averaging time is long compared with the ciprocal of the lowest frequency in the original signal, en repeated measurements of the product for a given τ will eld values very close to one another.

f. Periodic Functions

If $x_1(t)$ contains any periodic frequency components, then $\mathcal{R}(\tau)$ will contain each of these frequency components. So for certain time functions, $\mathcal{R}(\tau)$ is known if the iginal signal $x_1(t)$ is given.

g. Physical Processes

Any physical realizable process that produces time function like $x_1(t)$ is such that the value of the nction at time t becomes more independent of the value at $+ \tau$ as τ gets larger. Thermal noise, and even the quantum

mechanical uncertainty principle, introduce randomness that cause a gradual loss of coherence between the signal and its delayed replica. This is true even for the oscillators in the most stable atomic clock.

Hence for a signal arising from a real process the autocorrelation function approaches zero as τ becomes sufficiently large.

h. Coherence Time

The value of τ that causes a significant reduction in the autocorrelation function is a measure of the coherence time of the original signal $x_1(t)$.

i. The Frequency Domain

A less obvious property of the autocorrelation function is indicated by:

$$\mathcal{R}(\tau) \Longleftrightarrow \phi(\omega) \tag{7}$$

Equation (7) means that the autocorrelation function and the power spectral density are Fourier Transform pairs. Mathematically:

$$\phi(\omega) = \int_{-\infty}^{\infty} \mathcal{R}(\tau) e^{-j\omega\tau}\, d\tau \tag{8}$$

$$\mathcal{R}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(\omega) e^{j\omega\tau}\, d\omega \tag{9}$$

The power spectral density for deterministic case II signals is:

$$\phi(\omega) = \lim_{T\to\infty} \frac{1}{2T} X_{1T}(\omega)\ X_{1T}^{*}(\omega) \tag{10}$$

$$= \lim_{T\to\infty} \frac{1}{2T} |X_{1T}(\omega)|^2$$

where the subindex T indicates that the function $x_1(t)$ is truncated and hence observed through a window of observation of duration T.

The power spectral density for deterministic case I signals is:

$$\phi(\omega) = X_1(\omega)\ X_1^{*}(\omega) = |X_1(\omega)|^2 \tag{11}$$

The same property holds for stochastic processes, in which case:

$$R(\tau) \Leftrightarrow \phi(\omega) \tag{12}$$

and

$$\phi(\omega) = \lim_{T\to\infty} \frac{1}{2T} E\{|X_{1T}(\omega)|^2\} \tag{13}$$

j. White Noise

The autocorrelation function for very wideband, uniform (white) noise with r.m.s. value E_n is an impulse function at $\tau = 0$ with amplitude E_n^2. This means that one characteristic of such kinds of noise is that its instantaneous value is completely independent of the value at any other instant and that the coherence time of the process producing the noise is practically zero or very short.

B. CROSSCORRELATION FUNCTION

While the autocorrelation function of a signal is equivalent to the traditional technique of power density spectrum analysis, there is no classical analogy for crosscorrelation analysis. Crosscorrelation is concerned with the relationship between two different signals that arise in some common process [Ref. 3].

The mathematical expression of crosscorrelation is:

$$\mathcal{R}_{12}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x_1(t)\, x_2(t + \tau)\, dt \qquad (14)$$

1. Properties of the Crosscorrelation [Ref. 3]

The properties of the crosscorrelation function are, in general, quite different from those of the autocorrelation function.

a. Type of Function

The crosscorrelation function is not an even function of τ, which means:

$$\mathcal{R}_{12}(\tau) \neq \mathcal{R}_{12}(-\tau) \qquad (15)$$

However, $$\mathcal{R}_{12}(-\tau) = \mathcal{R}_{21}(\tau) \qquad (16)$$

a relationship that has practical importance in obtaining $\mathcal{R}_{12}(\tau)$ for negative delays.

b. Coherence

The crosscorrelation function approaches zero

as τ approaches infinity for signals that arise from real physical processes, due to the presence of noise and the uncertainty principle.

Furthermore if $x_1(t)$ and $x_2(t)$ arise from two completely separate, unrelated processes

$$\mathcal{R}_{12}(\tau) = 0 \tag{17}$$

c. Frequency Domain

The same as in the case of autocorrelation functions:

$$\mathcal{R}_{12}(\tau) \Longleftrightarrow \phi_{12}(\omega) \tag{18}$$

In this case, however, the physical meaning of $\phi_{12}(\omega)$ is not so clear. It can be called the spectrum of crosscorrelation of the time functions $x_1(t)$ and $x_2(t)$.

Figure 2 shows some typical examples of cross-correlation between functions.

Crosscorrelation analysis provides a powerful analytical tool. The ability to measure the degree to which the signals that arise from a common physical phenomenon resemble each other as a function of the delay time between them, can provide a much deeper insight into the phenomenon being studied than a separate analysis of the properties of either signal alone.

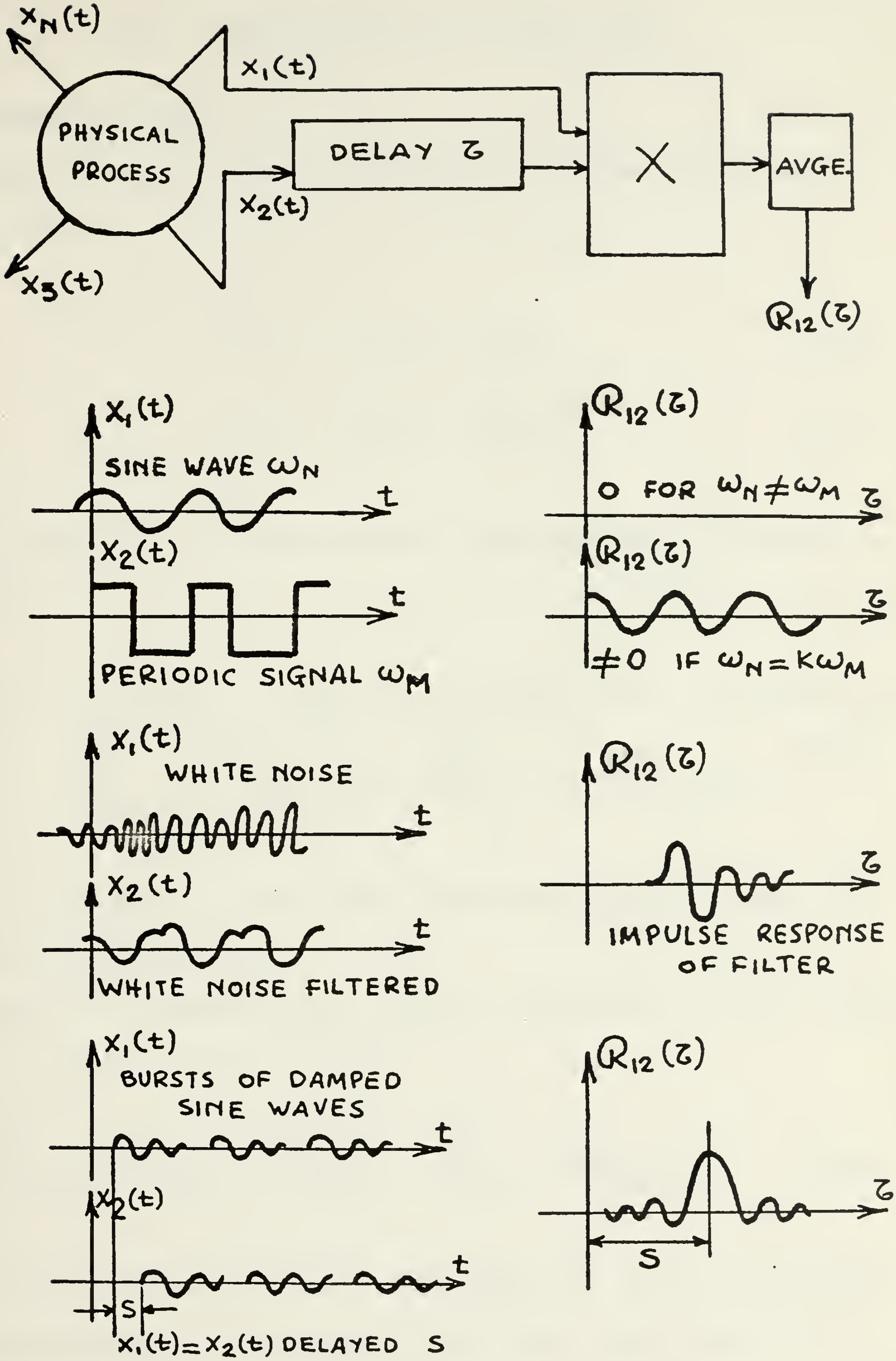


FIGURE 2: Typical Crosscorrelation Functions.

III. APPLICATIONS OF CORRELATION

A. DETECTION [REF. 4]

Given two functions consisting of a signal S(t) and noise N(t):

$$X_1(t) = S_1(t) + N_1(t) \tag{19}$$

$$X_2(t) = S_2(t) + N_2(t) \tag{20}$$

it is possible to crosscorrelate them with the following result:

$$\begin{aligned}\mathcal{R}_{12}(\tau) &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [S_1(t) + N_1(t)][S_2(t+\tau) + N_2(t+\tau)]\, dt \\ &= \mathcal{R}_{S_1S_2}(\tau) + \mathcal{R}_{S_1N_2}(\tau) + \mathcal{R}_{N_1S_2}(\tau) + \mathcal{R}_{N_1N_2}(\tau)\end{aligned} \tag{21}$$

All the terms of the above expression will be zero as τ approaches infinity.

Now it is assumed $X_1(t) = X_2(t)$ and hence the four terms of the crosscorrelation become

$$\mathcal{R}_{11}(\tau) = \mathcal{R}_{SS}(\tau) + \mathcal{R}_{SN}(\tau) + \mathcal{R}_{NS}(\tau) + \mathcal{R}_{NN}(\tau) \tag{22}$$

In the expression thus obtained $\mathcal{R}_{SS}(\tau)$ is the autocorrelation function of the signal itself and is a function of τ other than zero even for large values of τ. The remaining terms approach zero as τ becomes larger. So the first term

is a measure of the signal and the other ones are a measure of noise.

If now it is assumed that the frequency of the given signal is known, an internal generator in the receiver can be used to generate the second input to the correlator and the two signals to crosscorrelate are:

$$X_1(t) = S_1(t) + N(t) \tag{23}$$

$$X_2(t) = S_2(t) \tag{24}$$

The above one is the typical case of a radar receiver, and if the two signals are crosscorrelated the result is:

$$\mathcal{R}_{12}(\tau) = \mathcal{R}_{S_1S_2}(\tau) + \mathcal{R}_{NS_2}(\tau) \tag{25}$$

Now the number of noise terms has been reduced to one and it is reduced to zero for large values of τ. The previous examples show how correlation techniques can be used to separate desired signals from the noise.

B. ANALYSIS OF LINEAR SYSTEMS [REF. 3]

It can be shown that it is possible to obtain a unit impulse response of a linear system by driving it with broadband-white-noise and crosscorrelating this input with the system output, as is shown in Figure 3. The correlogram that results gives the same information about the system as if it were excited with an approximation to a delta function

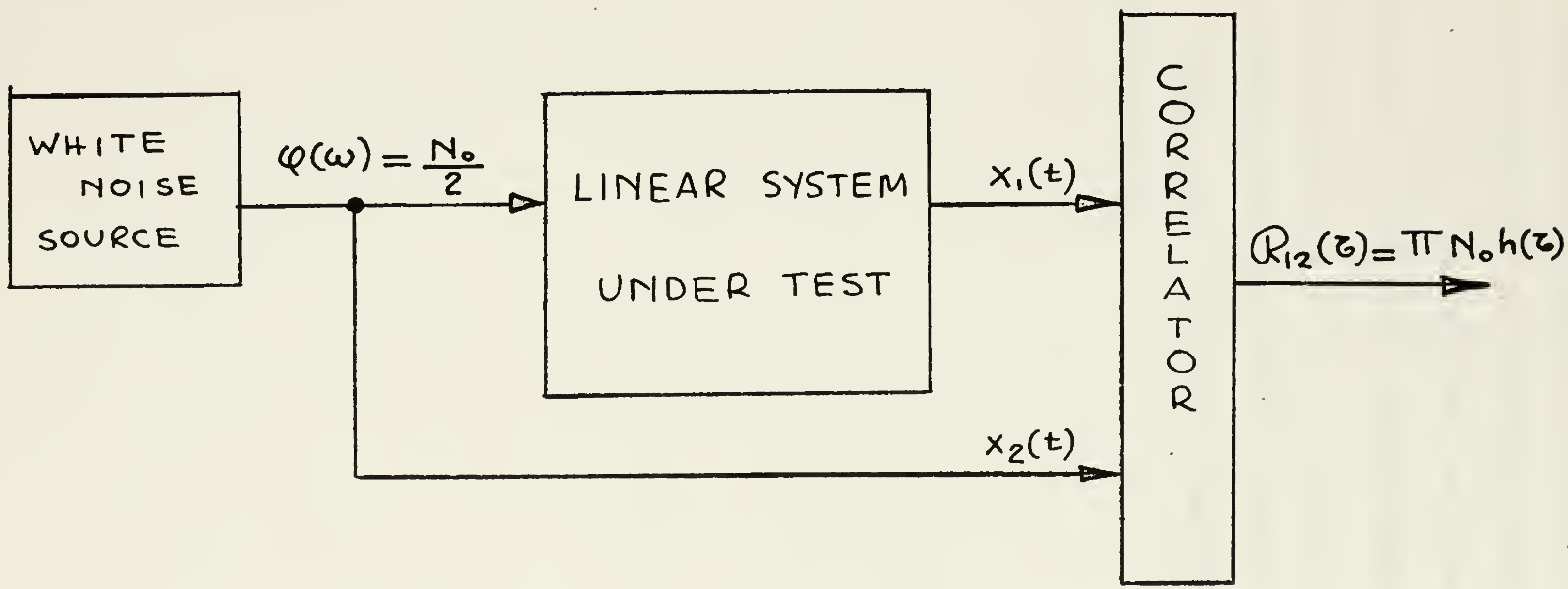


FIGURE 3: LINEAR SYSTEM ANALYSIS

and its output recorded on an oscilloscope or x-y recorder.

This technique has important practical applications because of its immunity to internal system noise and because it permits to obtain the impulse response of a linear system in the presence of a signal in the linear system.

With real time methods of correlation it is possible using the method described above to keep the response of critical systems under constant surveillance and to make optimizing adjustments.

IV. METHODS TO OBTAIN CORRELATION FUNCTIONS

There exist three methods to obtain correlation functions:

A. METHOD THROUGH DEFINITION-TIME DOMAIN

The easiest way to obtain a correlation seems to be the process indicated in its mathematical definition (1) and (14), this means: a delay τ_1 is given to one of the signals, then the other signal and the delayed one are multiplied and finally the product is averaged being the result $\mathcal{R}(\tau_1)$ one point of the correlation function. Varying τ it is possible to obtain a graph representing the correlation function.

A basic block diagram to perform the process was shown in Figures 1 and 2. The essential feature of such devices is the temporal delay τ between the two signals. When the signals are fluctuating voltages or currents, the provision of the "pure" delay is rather a problem because the available delay devices produce distortion on the signals for long delay times.

Another problem of this method is that it is essentially discrete, and the number of points of the correlation function that can be obtained in a given time interval depends upon the number of individual delay devices.

Generally to avoid distortion, digital delay devices are used [Ref. 3] and hence it is possible to obtain a number of discrete values of the correlation function without distortion.

Although this method is not cheap it is widely used for receivers which use correlator detectors.

B. INDIRECT METHOD-FREQUENCY DOMAIN

Equations (8), (9), (10), (11) and (13) set up the relations between correlation functions-time domain, and power spectral densities-frequency domain.

It is seen from the mentioned equations that correlation of two signals, or one signal with a shifted version of itself, in the time domain is equivalent to a complex conjugate multiplication of their linear spectra in the frequency domain.

In other words, the autospectrum and the autocorrelation are related through the Fourier Transform, the correlation being the inverse transform of the power spectrum. The same relation applies to the cross-spectrum and crosscorrelation.

Although this method is the faster one it is expensive because to obtain correlations, means have to be provided to obtain the power spectrums of the signals. Hence this method is typical of special signal processing machines which are designed to observe several characteristics of signals, and the power spectrums are not only used to obtain correlation functions.

C. METHOD WITHOUT "PURE" DELAY-TIME DOMAIN

This method has the advantage that a "pure" delay is not required. The correlation is obtained as a summation of

terms and the method is based upon the properties of orthogonal functions [Ref. 2].

This is the only method essentially analogous and as will be shown in the following sections inexpensive correlators can be built for some special kind of signals.

V. CORRELATION WITHOUT A "PURE" TIME DELAY

A. MATHEMATICAL FORMULATION

1. Analysis Problem

It is supposed that any correlation function $\mathcal{R}(\tau)$, either autocorrelation or crosscorrelation, can be expanded in a series of orthogonal functions [Ref. 2], thus

$$\mathcal{R}(\tau) = \sum_{n=0}^{\infty} a_n \theta_n(\tau)[\omega(\tau)]^{\gamma} \quad 0 \leq \tau < \infty \tag{26}$$

where the $\theta_n(\tau)$ are polynomials which form an othonormal set with respect to the weight function $\omega(\tau)$ in the range $0 \leq \tau < \infty$. Then the coefficients, a_n, in the series expansion can be found in the usual way and are given by

$$a_n = \int_0^{\infty} \mathcal{R}(\tau) \theta_n(\tau)[\omega(\tau)]^{1-\gamma} d\tau \tag{27}$$

Now the system of Figure 4 is considered. The linear network has an impulse response $h_n(t)$. So the filter output is:

$$x_1(n;t) = \int_0^{\infty} x_1(t-u) h_n(u) du \tag{28}$$

and the multiplier output is

$$x_1(n;t) x_2(t) = \int_0^{\infty} x_1(t-u) x_2(t) h_n(u) du \tag{29}$$

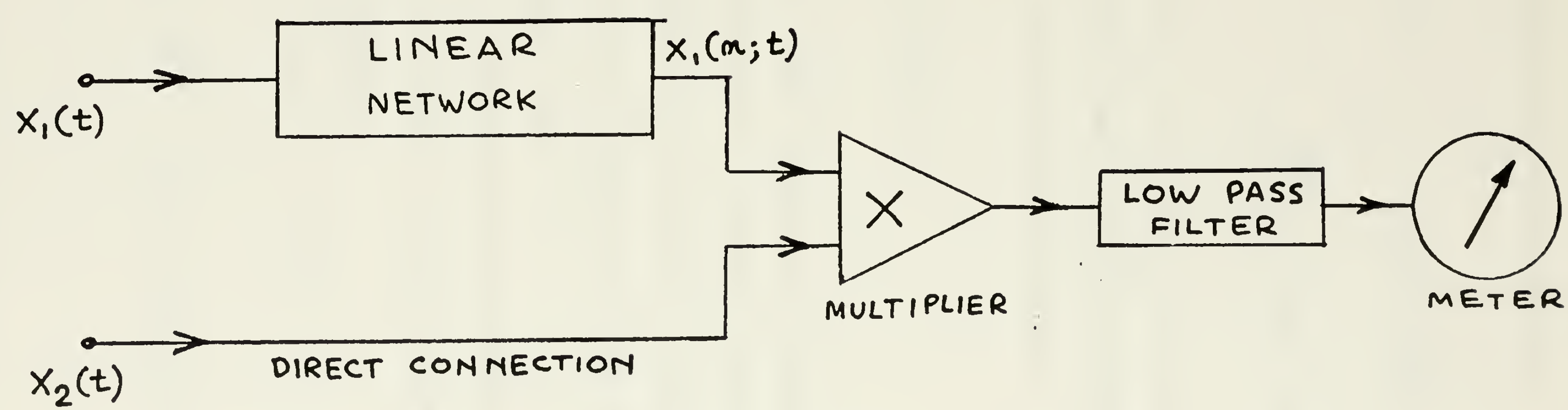


FIGURE 4: OBTENTION OF COEFFICIENTS

being the output of the averaging device

$$\overline{x_1(n;t)\ x_2(t)} = \int_0^{\infty} \overline{x_1(t-u)\ x_2(t)}\ h_n(u)\ du \tag{30}$$

$$= \int_0^{\infty} \mathcal{R}(u)\ h_n(u)\ du \tag{31}$$

Now comparing (31) to (27) it is seen that the coefficient a_n is identical with the average $\overline{x_1(n;t)\ x_2(t)}$ provided that the filter impulse response is so chosen that

$$h_n(t) = \theta_n(t)[\omega(t)]^{1-\gamma} \tag{32}$$

The filters whose impulse responses satisfy Equation (32) are called "orthogonal filters," and if they could be built, the coefficients a_n, would be obtained simply by switching in each of the filters in turn and determining the corresponding averages [Ref. 2].

Naturally, in any practical system only a finite number, say (N+1), of filters would be used.

2. Synthesis Problem

Equation (26) can be written

$$\mathcal{R}(\tau) = [\omega(\tau)]^{2\gamma-1} \sum_{n=0}^{\infty} a_n\ \theta_n(\tau)[\omega(\tau)]^{1-\gamma} \tag{33}$$

or

$$\mathcal{R}(t) = [\omega(t)]^{2\gamma-1} \sum_{n=0}^{\infty} a_n\ h_n(t) \quad 0 \leq t < \infty \tag{34}$$

In particular, if a finite number of terms are taken, say N, it can be written as an approximation to (33)

$$\mathcal{R}_N(t) = [\omega(t)]^{2\gamma-1} \sum_{n=0}^{N} a_n h_n(t) \tag{35}$$

If γ is taken to be zero, (35) becomes

$$\mathcal{R}_N(t) = [\omega(t)]^{-1} \sum_{n=0}^{N} a_n h_n(t) \tag{35a}$$

and if γ is taken to be 1/2

$$\mathcal{R}_N(t) = \sum_{n=0}^{N} a_n h_n(t) \tag{35b}$$

Figure 5 shows the complete synthesis of the method. It is possible to use half the number of linear networks calculating first the coefficients due to the particular signals to be correlated, and then after introducing the values of a_n the approximate correlation function can be seen on the screen of a CRT as a transient following the application of the input impulse [Ref. 2].

A convenient system results if in fact the pulsing can be carried out repetitively at intervals that are long compared with the filter time constants, and yet sufficiently frequently to give a steady trace on the CRT screen. This can always be arranged in practice [Ref. 2].

It can be seen from Equation (35a) that if γ is taken to be zero, the output of the circuit in Figure 5 has

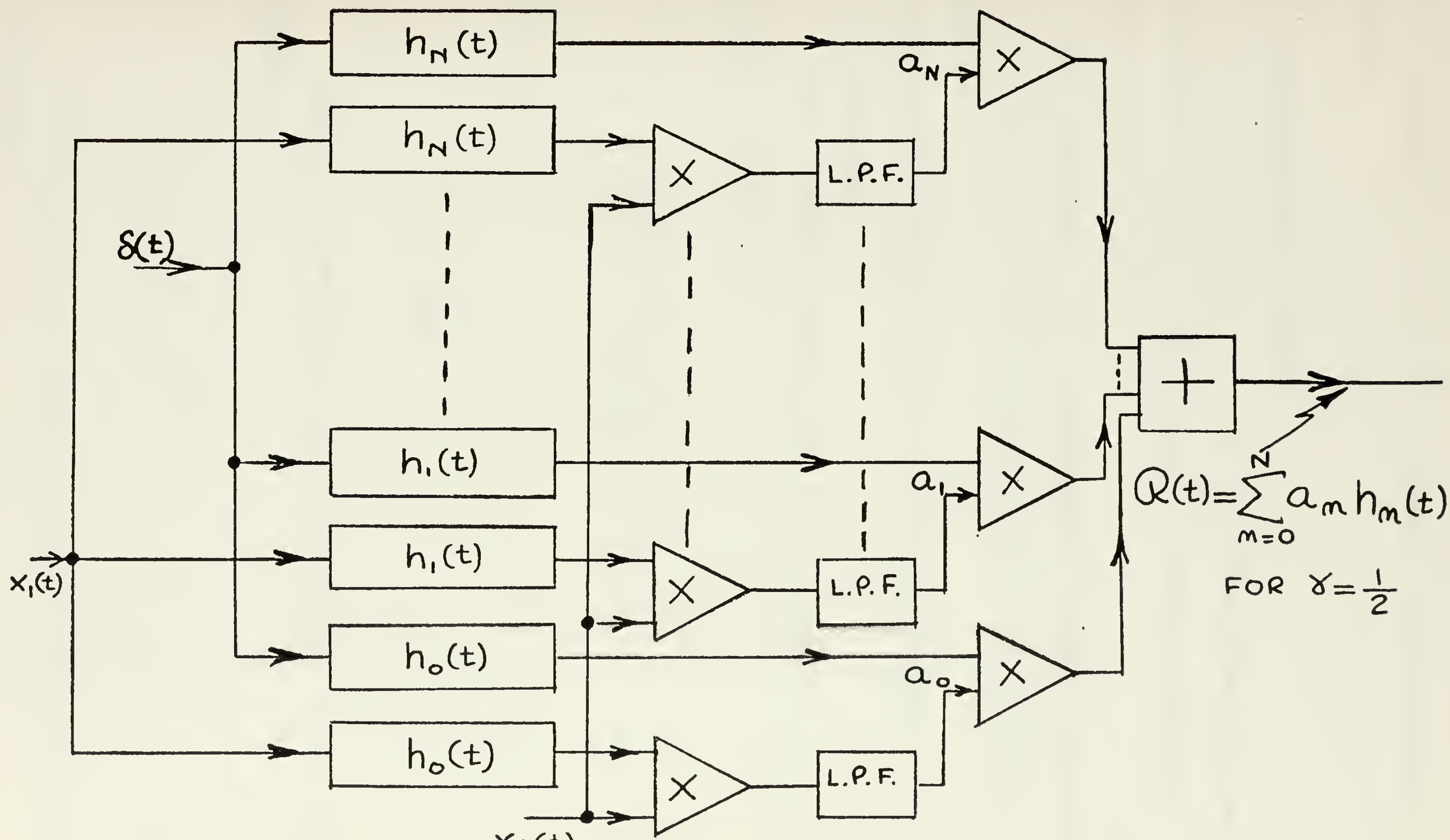


FIGURE 5: METHOD WITHOUT "PURE" DELAY

to be multiplied by $[\omega(t)]^{-1}$ to obtain the correlation function.

B. TWO REALIZABLE SYSTEMS

The applicability of all the preceding theory depends on realizing a set of filters whose impulse responses satisfy Equation (32).

The polynomials $\theta_n(t)$ and hence the impulse responses $h_n(t)$ depend only on the weight function $\omega(t)$ and the parameter γ.

The weight function $\omega(t) = \alpha\, e^{-\alpha t}$ was chosen.

1. <u>Case of $\gamma = 0$</u>

The transfer function H(s) of a single R-C cell (low pass filter) is:

$$H_{RC}(s) = \frac{1}{\tau}\,\frac{1}{s + \frac{1}{\tau}} = \frac{\alpha}{s + \alpha} \tag{36}$$

being

$$\alpha = \frac{1}{\tau} = \frac{1}{RC}$$

$H_{RC}(s)$ corresponds in the time domain to the impulse response:

$$h_{RC}(t) = \alpha\, e^{-\alpha t} \tag{37}$$

If now H(s) of a single C-R cell (high pass filter) is considered:

$$H_{CR}(s) = \frac{s}{s + \frac{1}{\tau}} = \frac{s}{s + \alpha} \tag{38}$$

and the impulse response in the time domain is:

$$h_{CR}(t) = -\alpha\ e^{-\alpha t} \tag{39}$$

Now a number n of high pass filter cells are cascaded and between each cell a buffer amplifier is connected. Buffer amplifiers have ideally infinite input impedance, zero output impedance and unity gain, and they isolate the cells.

Hence the transfer function in the s domain for the system of n high pass filter type cells is:

$$H_{nCR}(s) = [H_{CR}(s)]^n = \frac{s^n}{(s + \alpha)^n} \tag{40}$$

If a single low pass filter type cell is connected to the input of the circuit considered above via a buffer amplifier, the resultant circuit is of the form shown in Figure 6 and its transfer function in the s domain is:

$$H_n(s) = H_{RC}(s)\ H_{nCR}(s) = \frac{\alpha}{s + \alpha}\ \frac{s^n}{(s + \alpha)^n} \tag{41}$$

$$= \frac{\alpha\ s^n}{(s + \alpha)^{n+1}} \tag{42}$$

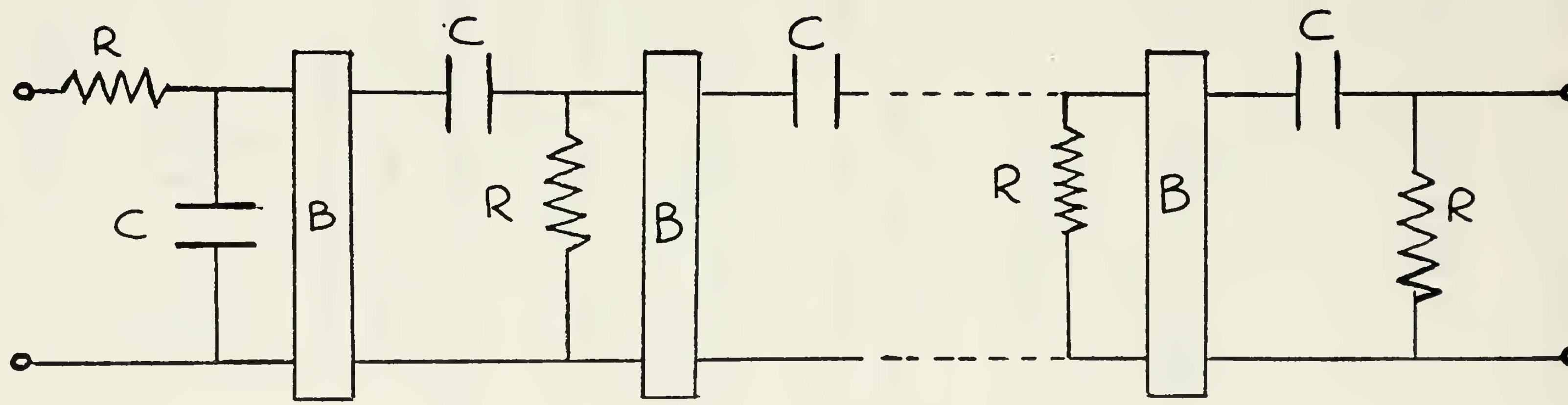


FIGURE 6: LAGUERRE FUNCTION TYPE CIRCUIT $\gamma = 0$

The inverse Laplace transform of the above one is:

$$\alpha \frac{d^n}{dt^n} \frac{1}{n!} t^n e^{-\alpha t} \tag{43}$$

which can be written as:

$$h_n(t) = \alpha e^{-\alpha t} \sum_{k=0}^{n} \frac{n!}{(n-k)!} \frac{(-\alpha t)^k}{k!\ k!} \tag{44}$$

$$= \alpha e^{-\alpha t} L_n(\alpha t)$$

where $\alpha e^{-\alpha t}$ is the weight function and $L_n(\alpha t)$ are the Laguerre polynomials and the product is defined as Laguerre functions.

The given expression for $h_n(t)$ satisfies Equation (32) and the conditions of orthogonality [Ref. 2] and hence the circuit of Figure 6 can be used to build the correlator.

2. Case of $\gamma = 1/2$

Another filter based on the same weight function is the lattice structure shown in Figure 7. In this case the s domain transfer function is given by

$$H_n(s) = \frac{\frac{\alpha}{2}}{s + \frac{\alpha}{2}} \left(\frac{s - \frac{\alpha}{2}}{s + \frac{\alpha}{2}}\right)^n \tag{45}$$

and the impulse response is:

$$h_n(t) = \frac{\alpha}{2} e^{-\frac{\alpha}{2}t} L_n(\alpha t) \tag{46}$$

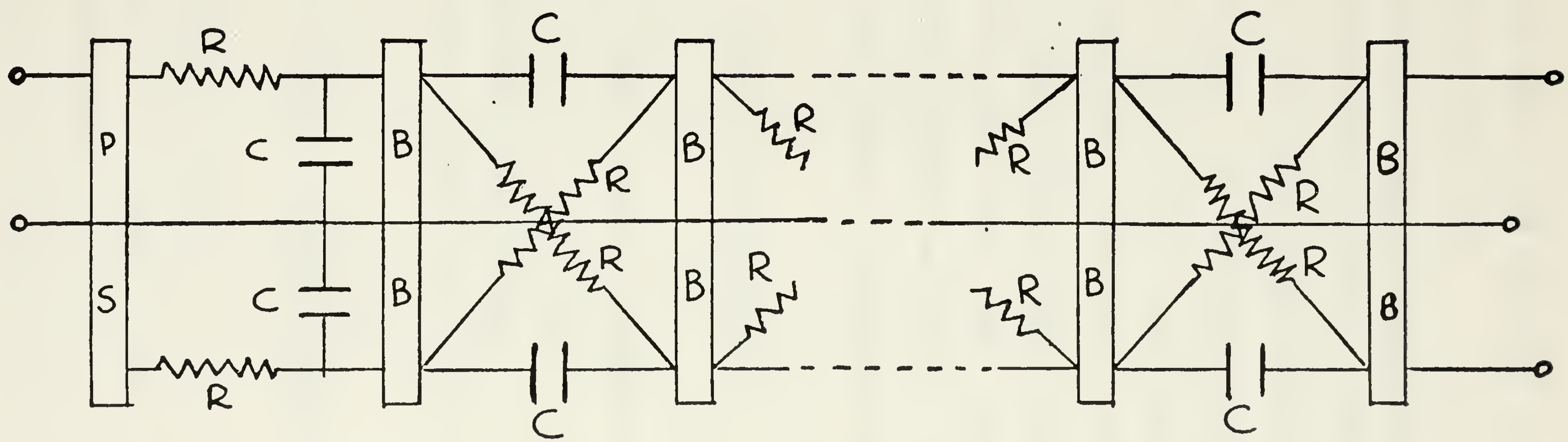


FIGURE 7: LAGUERRE FUNCTION TYPE CIRCUIT $\gamma = \frac{1}{2}$

which is identical as the case of $\gamma = 0$ except for a constant factor [Ref. 2].

3. Frequency Domain Considerations

The nonsymmetric filter, case of $\Upsilon = 0$, was used with good results to build an autocorrelator [Ref. 2] which used $N = 10$. Hence $N+1 = 11$ filters were used, being the first one a low pass filter ($n = 0$) and the rest combinations of a low pass filter and a different number of high pass filters isolated by cathode follower amplifiers.

We saw in the ideal case, which corresponds to perfect isolation buffer amplifiers, that the transfer function in the frequency domain was (41):

$$H(s) = \frac{\alpha}{s + \alpha} \left(\frac{s}{s + \alpha}\right)^n \tag{47}$$

and because of the isolation between cells it was taken as the product of the transfer function of a low pass filter and n R-C high pass filter cells.

For the $j\omega$ domain the above one becomes:

$$|H(j\omega)|^2 = \frac{\alpha^2 \omega^{2n}}{(\alpha^2 + \omega^2)^{n+1}} \tag{48}$$

being a function of α and n.

Figures 8, 9, 10 and 11 show the transfer functions in the frequency domain of a R-C low pass filter, an R-C high pass filter, ten high pass R-C cells cascaded and

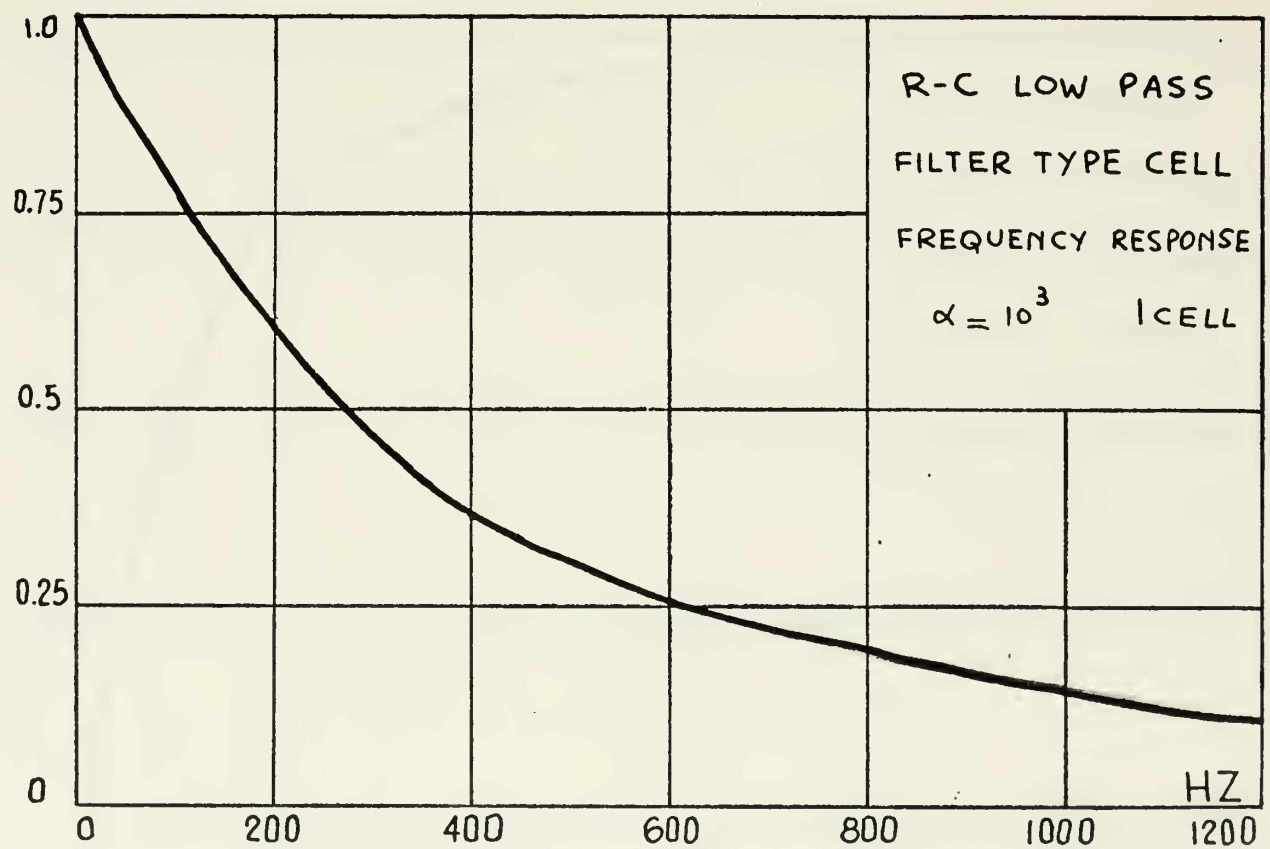

R-C LOW PASS
FILTER TYPE CELL
FREQUENCY RESPONSE
$\alpha = 10^3$ 1 CELL
1.0
0.75
0.5
0.25
0
0
200
400
600
800
1000
1200
HZ

FIGURE 8

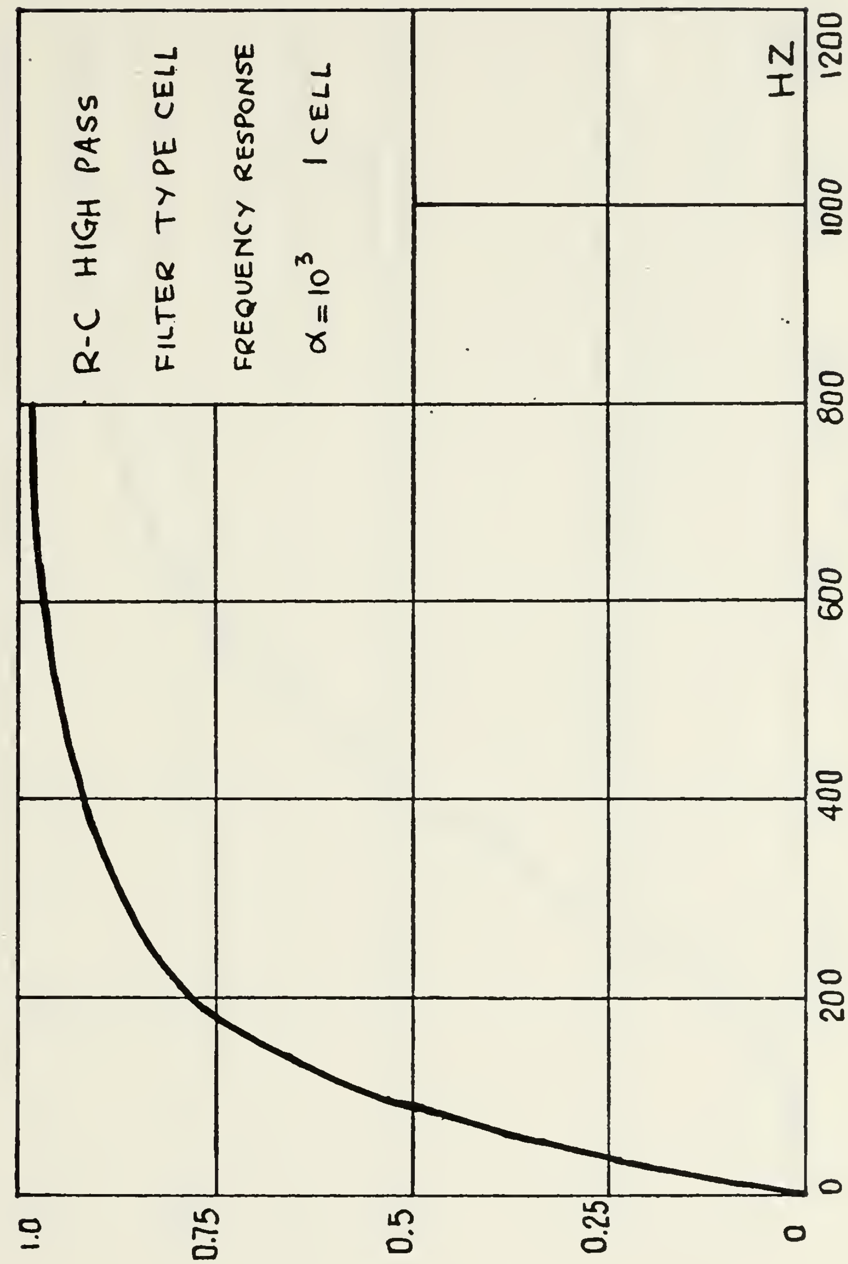

R-C HIGH PASS
FILTER TYPE CELL
FREQUENCY RESPONSE
$\alpha = 10^3$ 1 CELL
HZ
1.0
0.75
0.5
0.25
0
0
200
400
600
800
1000
1200

FIGURE 9

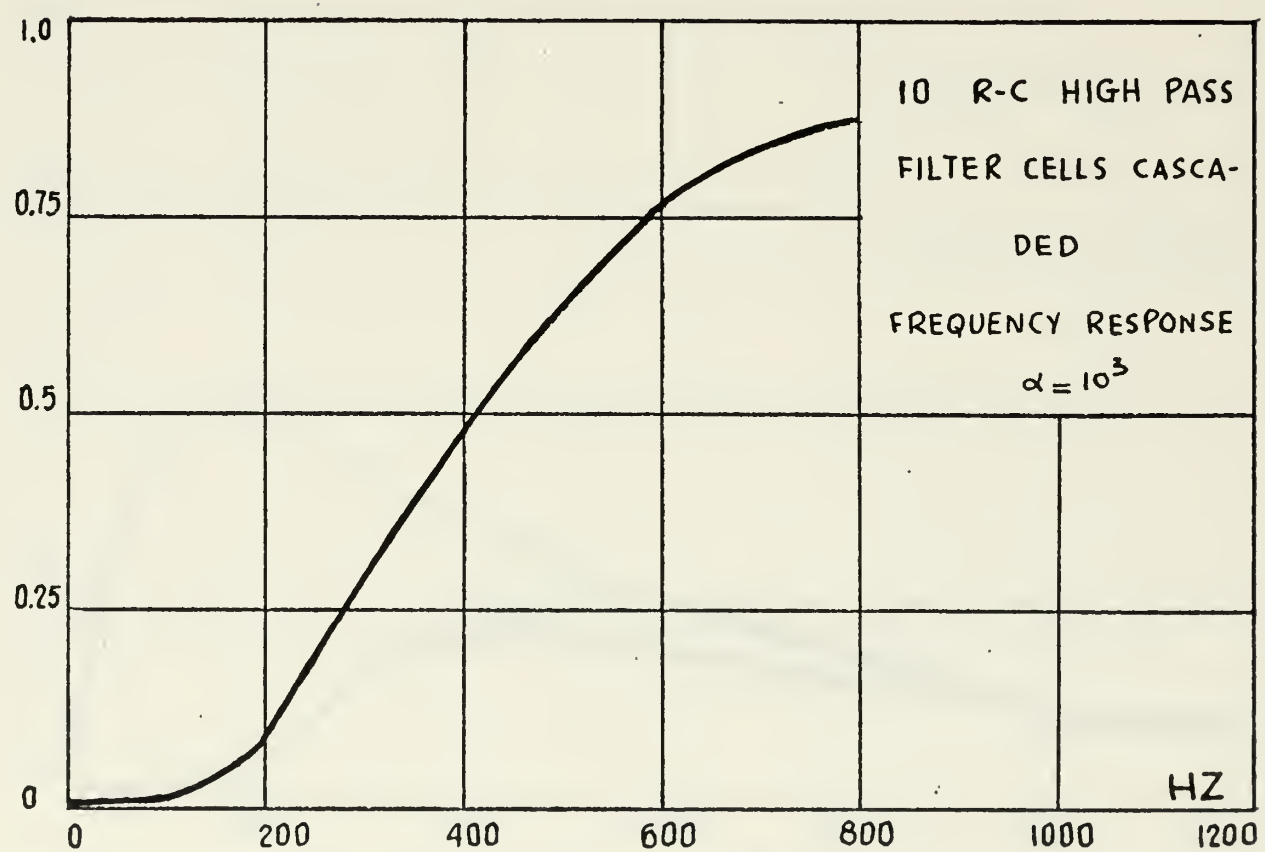

10 R-C HIGH PASS
FILTER CELLS CASCA-
DED
FREQUENCY RESPONSE
$\alpha = 10^3$
1.0
0.75
0.5
0.25
0
200
400
600
800
1000
1200
HZ

FIGURE 10

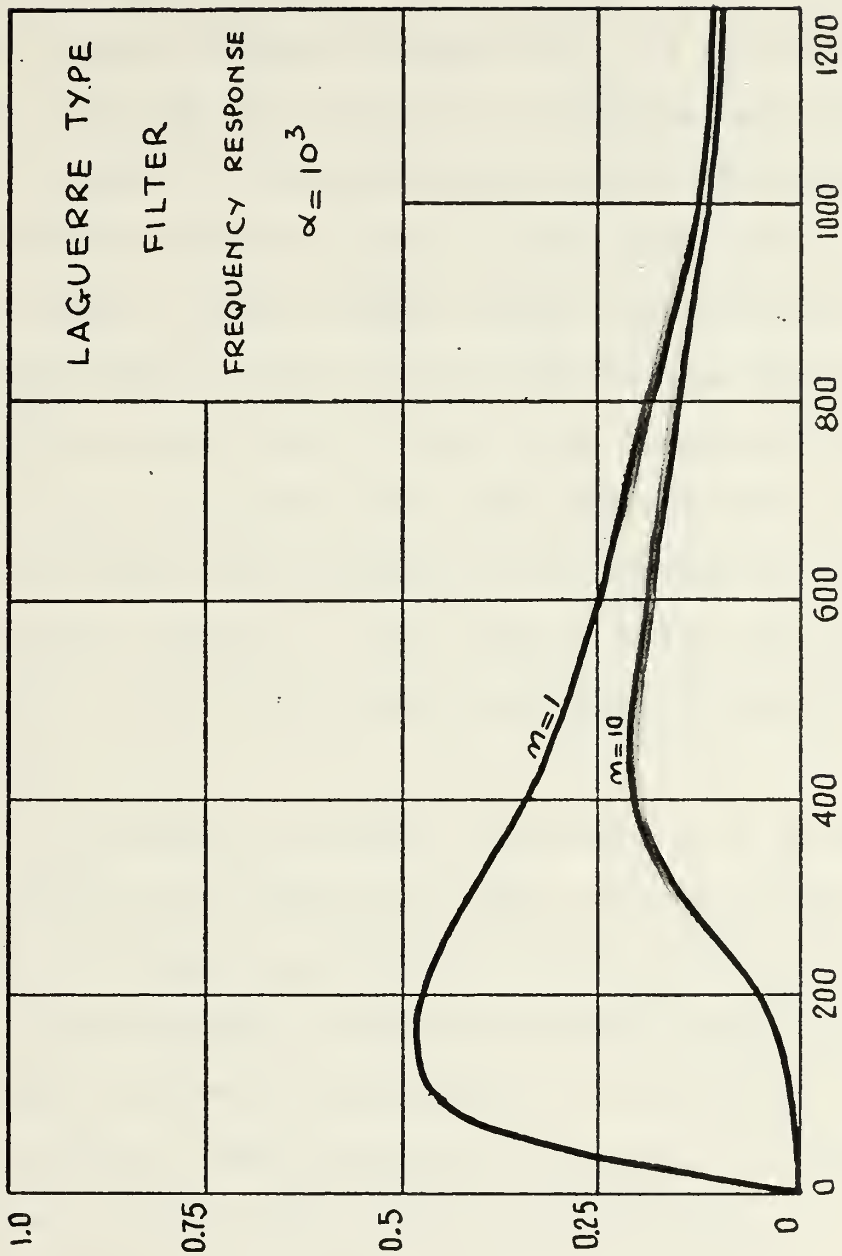

LAGUERRE TYPE
FILTER
FREQUENCY RESPONSE
$\alpha = 10^3$
m=1
m=10
1.0
0.75
0.5
0.25
0
0
200
400
600
800
1000
1200

FIGURE 11

ideally isolated, and of the ideal filters for n = 1 and n = 10, respectively. In all cases $\alpha = 10^3$ was taken.

The transfer functions were obtained with the aid of the CORNAP circuit analysis program [Ref. 1] for the case of individual cells and for the case of multiple cells cascaded taking the product of the individual transfer functions.

From the curves it can be easily seen which is the frequency range for the incoming signals and which the bandwidth of the system. Also if the incoming signals are given it is possible to see which filters need postamplification.

It has to be pointed out that the frequency domain considerations are only important in the determination of the coefficients (Figure 5) only case in which one of the input signals is convoluted with the impulse responses of the filters.

As the system is linear, by varying α it is possible to obtain filters for different ranges of frequencies.

The correlator described in Ref. 2 used $\alpha = 0$ type filters to autocorrelate a sine wave which frequency was 152 Hz, being the basic time-constant of the filter $\alpha = 843$ sec^{-1}. The results were practically identical to those which can be obtained with conventional methods.

C. A SET OF PRACTICAL FILTERS

The buffer amplifiers needed to build the filters have to have infinite input resistance, zero output resistence and unity gain and the system mentioned in Ref. 2 used

cathode followers as buffer amplifiers. A way to avoid the use of such amplifiers was studied in this work.

1. Factor of Isolation

A filter of multiple cells can be constructed in such a way that the time constant is maintained the same through the cells and the resistance is multiplied by a factor of k from cell to cell as in Figure 12. It can be shown that this is equivalent to a certain amount of isolation between cells which depends on the value of the mentioned factor:

$$\frac{R_2}{R_1} = \frac{k\ R_1}{R_1} = k = \text{Factor of Isolation} \qquad (49)$$

The maximum value of k that can be used depends upon the necessary number of cells, and the values of the capacitors and resistors available.

2. Computer Analysis

In the Appendix a program is given which allows the calculation of the Laguerre functions for any values of α, n and t. Figures 13 through 23 show the Laguerre functions for $\alpha = 10^3$ and for orders 0 up to 10.

The CORNAP circuit analysis program was used to obtain the impulse responses of circuits of several number of cells and different values of the factor of isolation.

Impulse responses of circuits up to seven cells (n=6) were obtained because the results had considerable errors for circuits with a large number of cells.

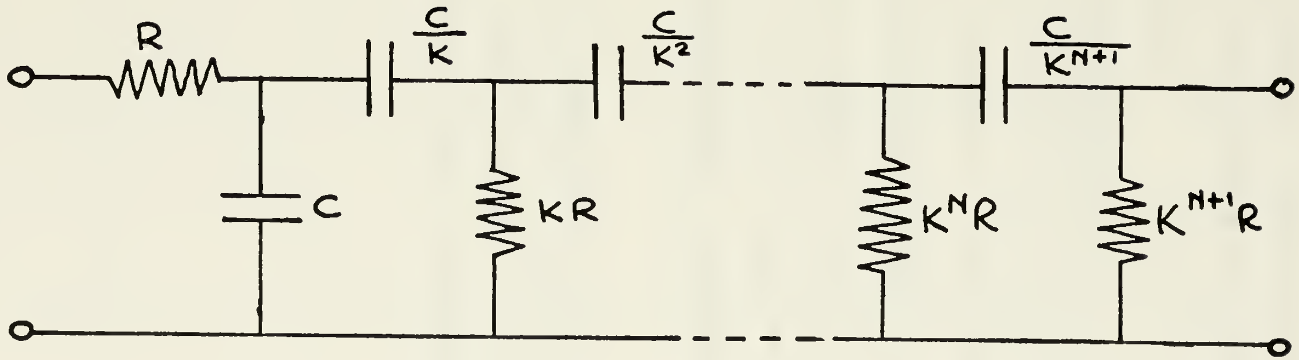


FIGURE 12: CIRCUIT WITH FACTOR OF ISOLATION = K

The results showed that circuits with a factor of isolation of 10 have exactly the same impulse response that the ideal ones. The lowest factor of isolation that gave reasonable approximation to the ideal impulse responses was found to be 5. Hence, a factor of isolation of 10 can be used whenever it is possible and the use of values less than 5 should be avoided.

3. Laboratory Results

Figures 24 and 25 show pictures of impulse responses of two kinds of circuits: first for $\alpha = 10^3$ and a factor of isolation of 10 for orders 0 up to 3, and second for the same α and k=5 for orders 0 up to 5.

It can be seen that the practical results obtained agree with the theoretical considerations and computer predictions.

It was not possible to build circuits with a larger number of cells because of the characteristics of the available pulse generator, oscilloscope and components.

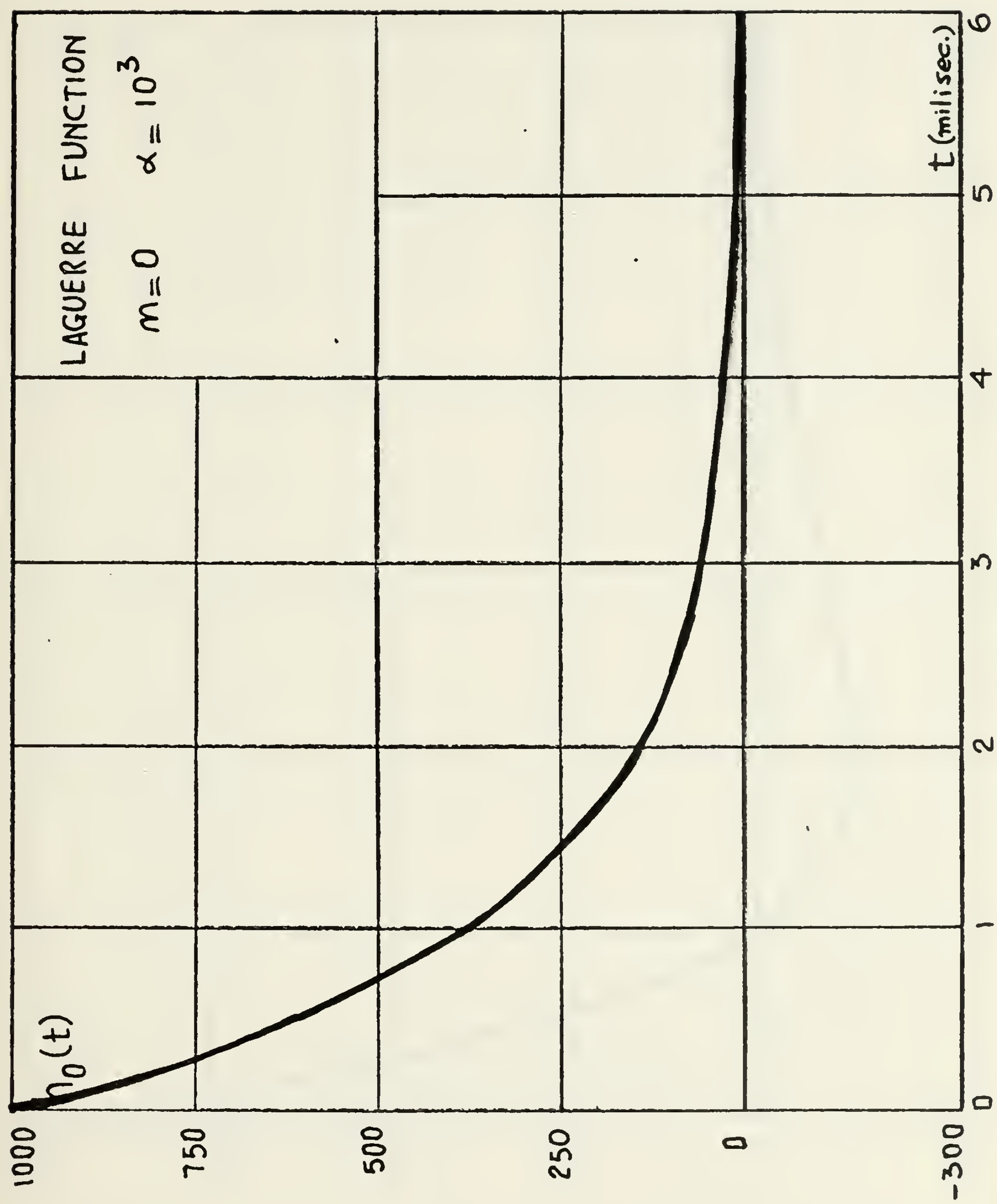

LAGUERRE FUNCTION
n=0 α=10^3
h_0(t)
t (milisec.)
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 13

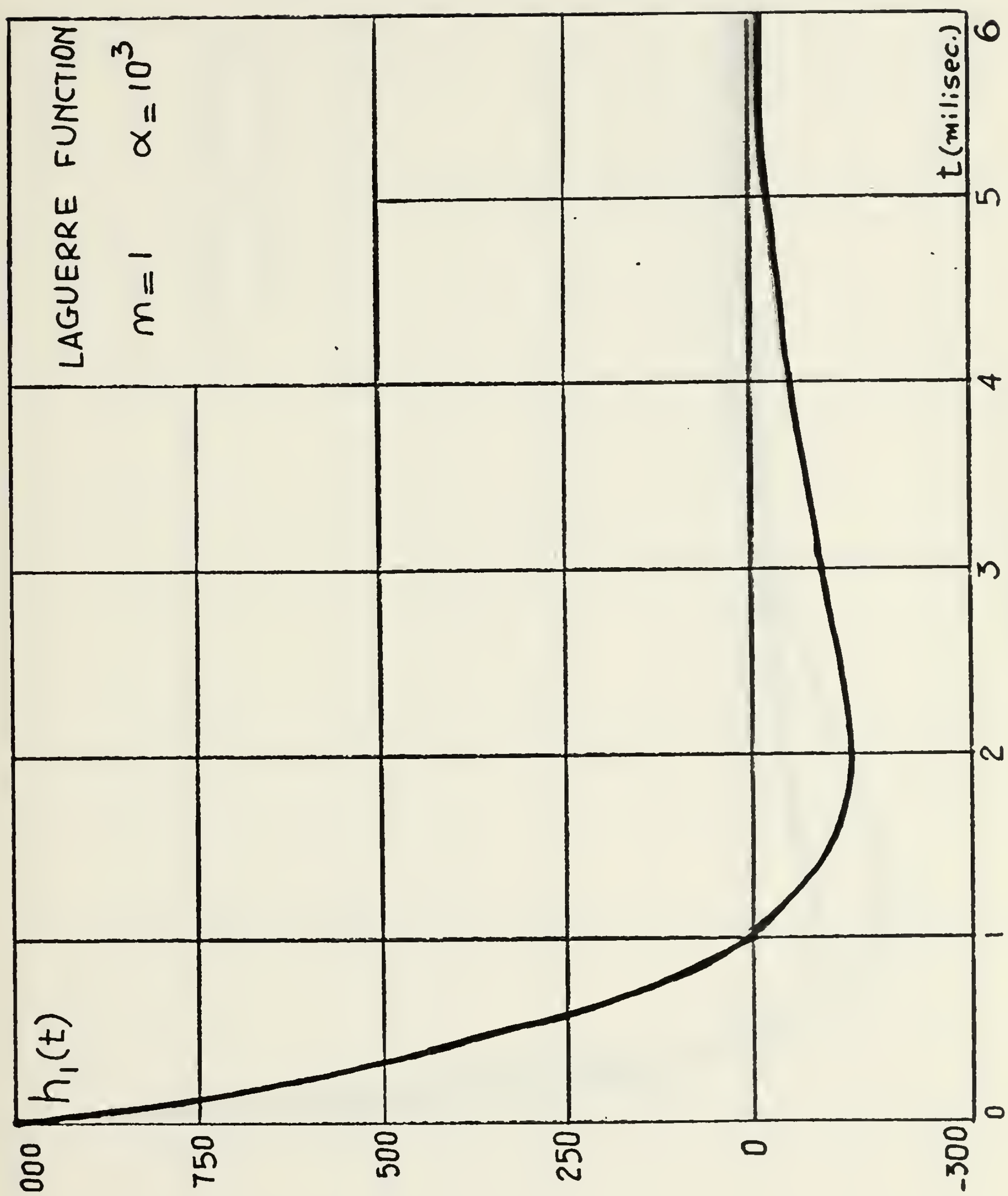

LAGUERRE FUNCTION
m=1
α=10^3
h_1(t)
t (milisec.)
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 14

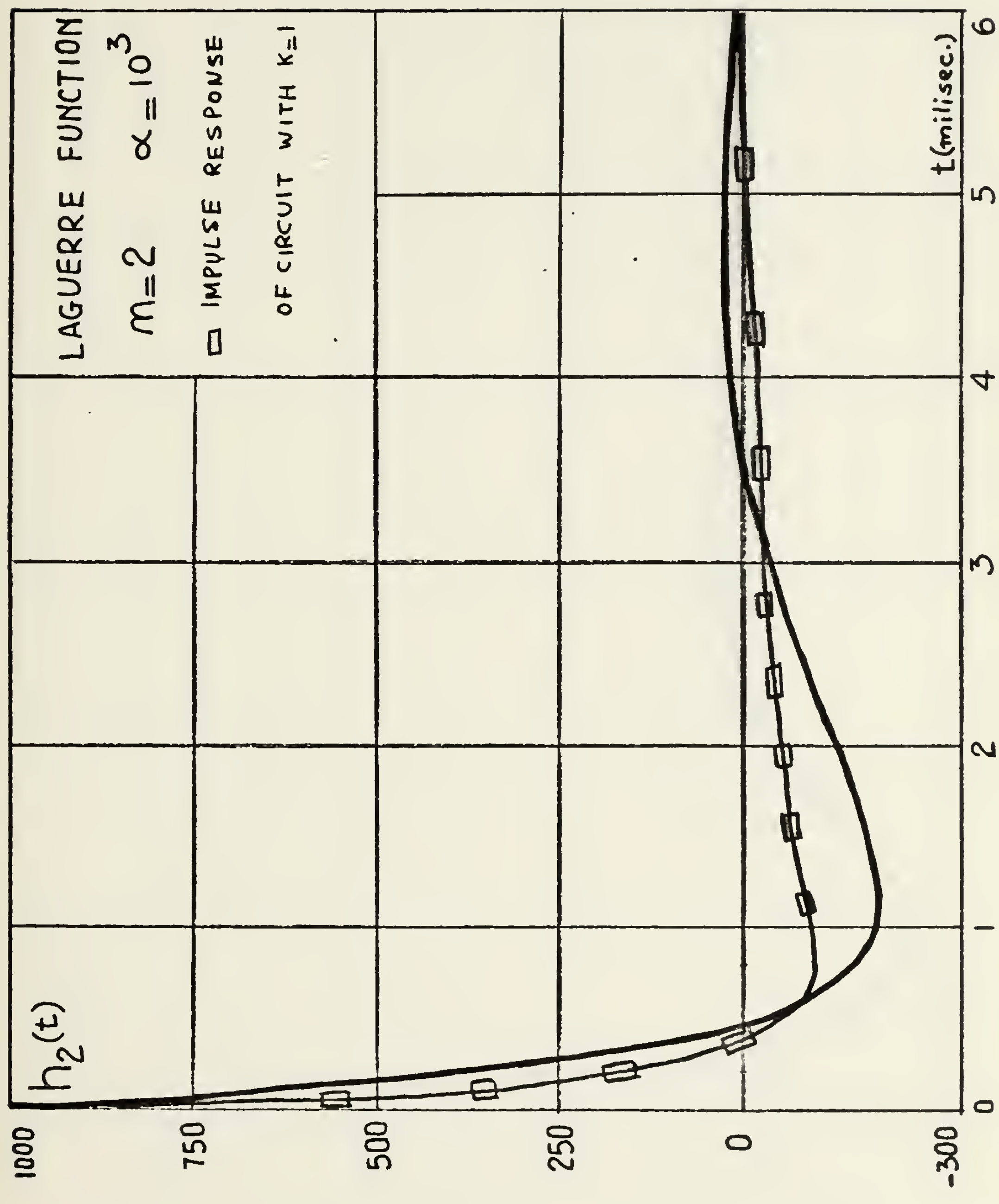
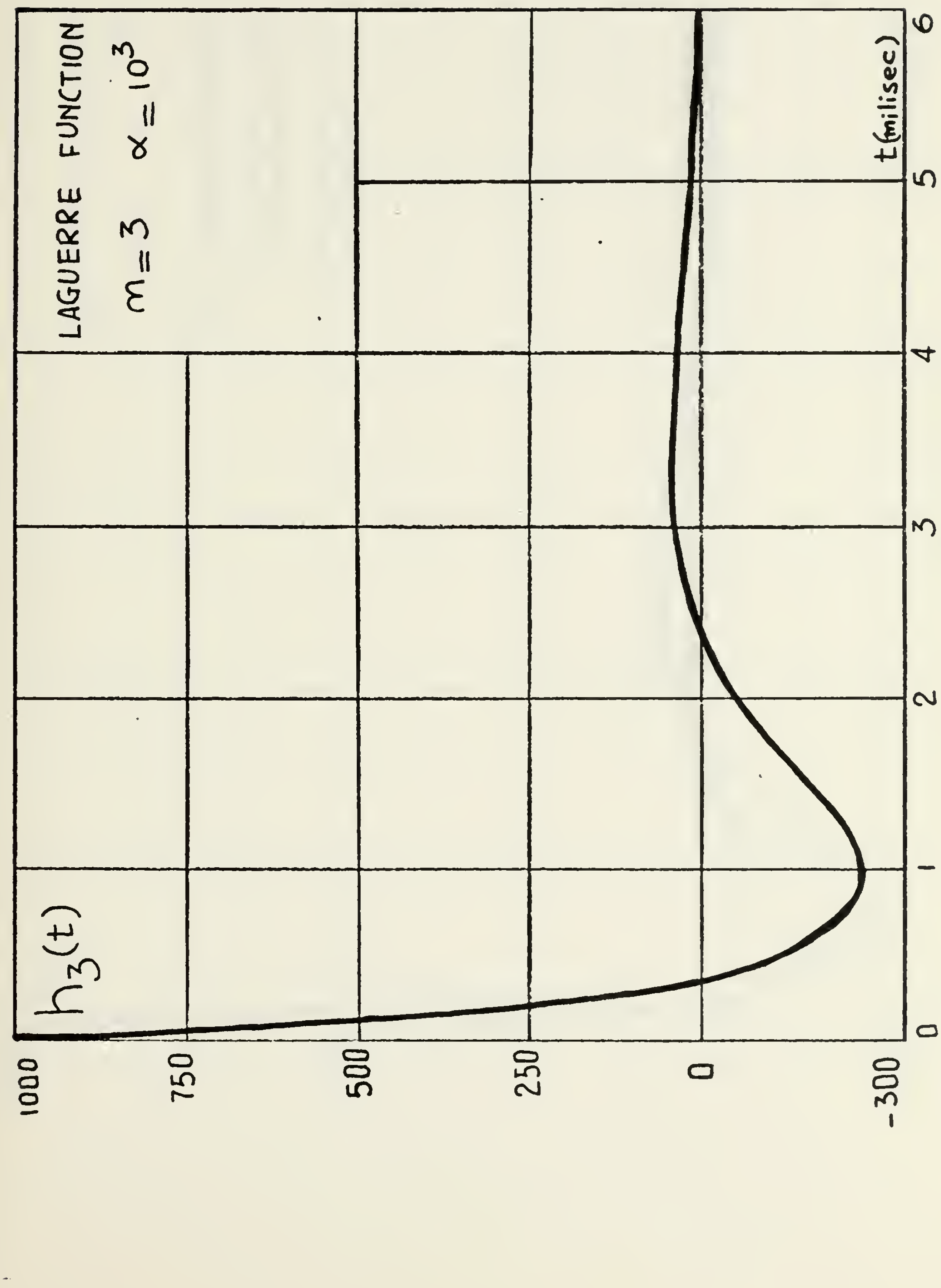
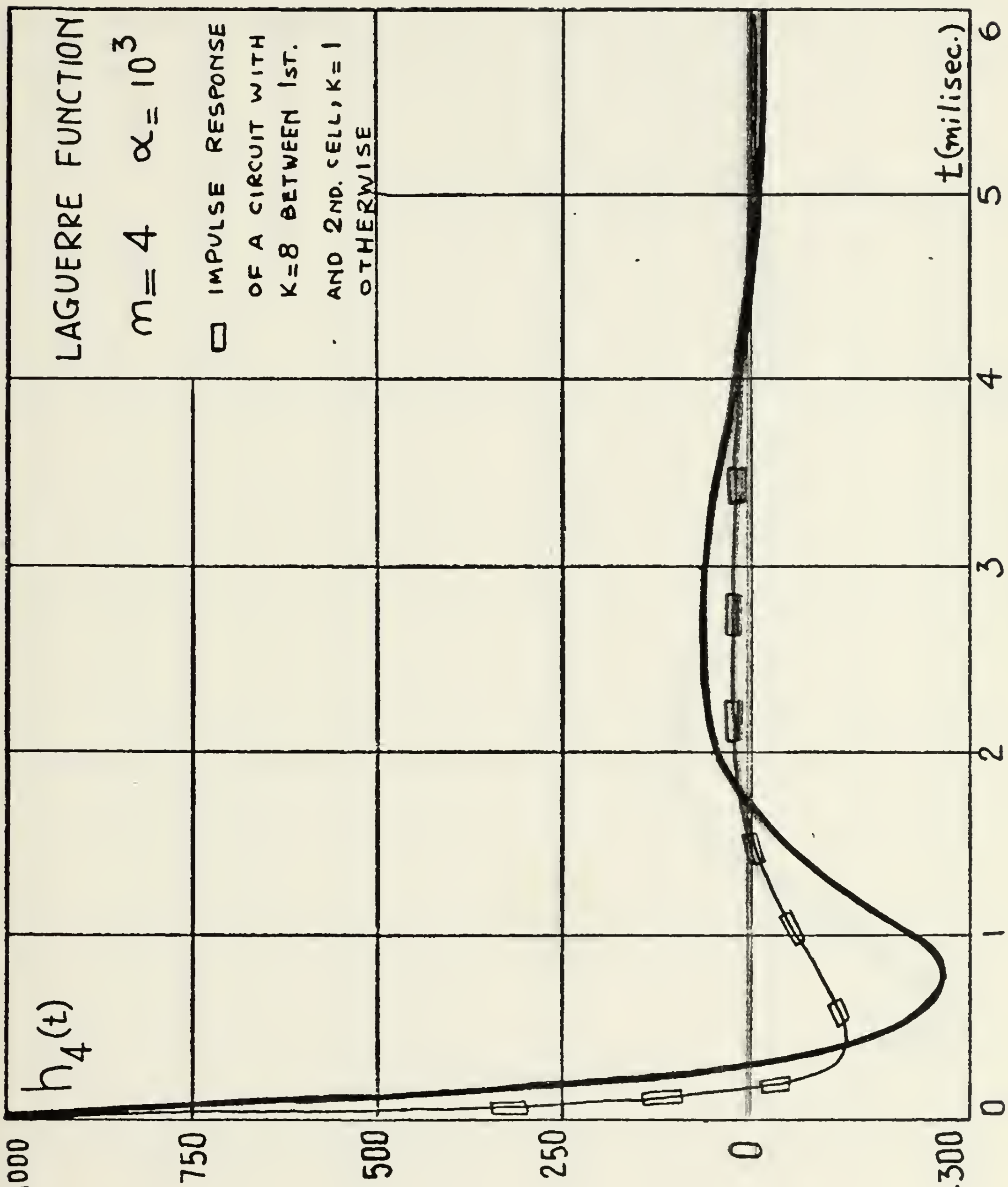

LAGUERRE FUNCTION
m=2 α=10^3
□ IMPULSE RESPONSE
OF CIRCUIT WITH K=1
h_2(t)
t(milisec.)
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 15



LAGUERRE FUNCTION
m = 3 α = 10^3
h_3(t)
1000
750
500
250
0
-300
0
1
2
3
4
5
6
t(milisec)

FIGURE 16



h_4(t)
LAGUERRE FUNCTION
m = 4 α = 10^3
□ IMPULSE RESPONSE OF A CIRCUIT WITH K=8 BETWEEN 1ST. AND 2ND. CELL, K=1 OTHERWISE
t (milisec.)
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 17

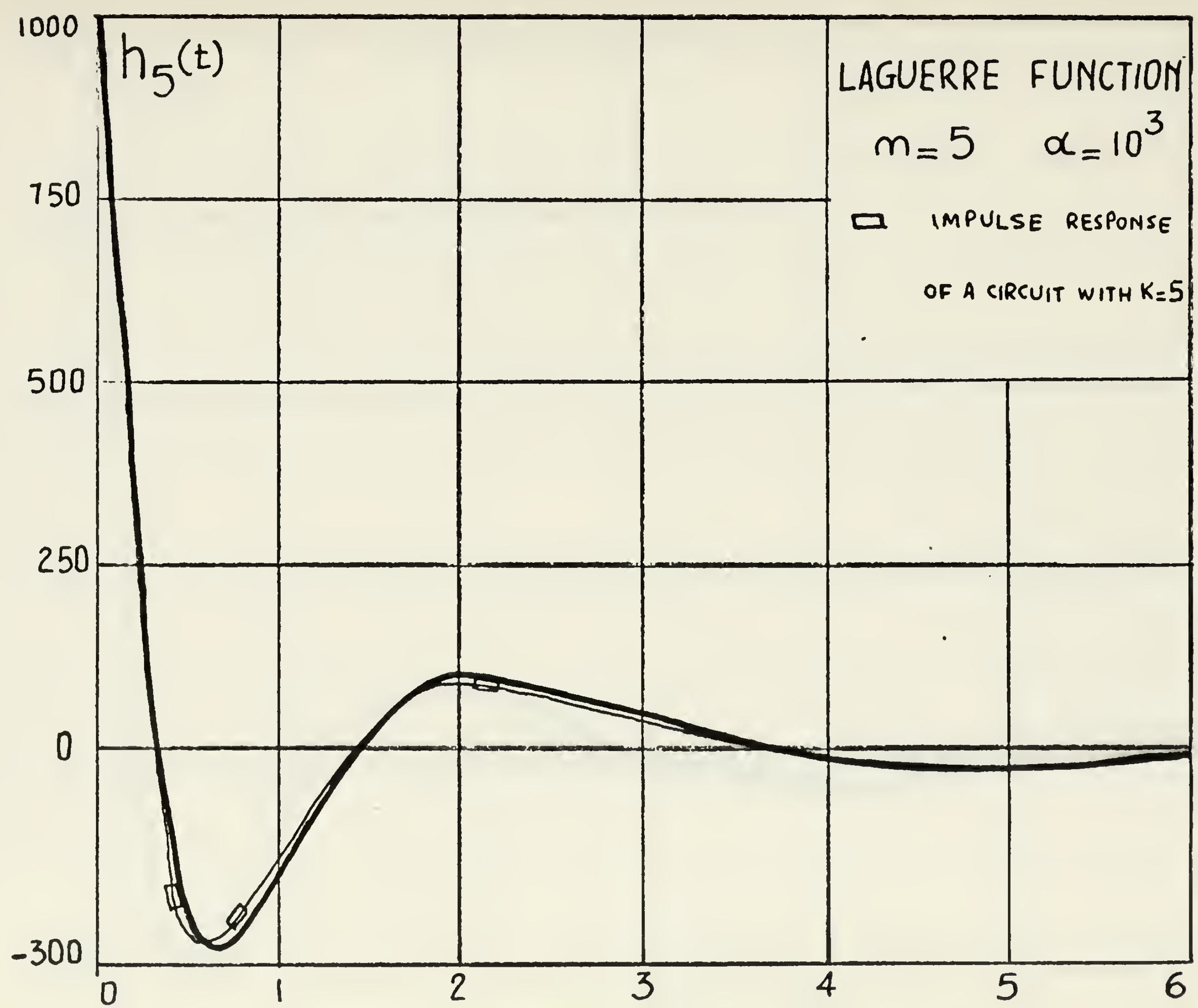

1000
750
500
250
0
-300
0
1
2
3
4
5
6
$h_5(t)$
LAGUERRE FUNCTION
$m = 5 \quad \alpha = 10^3$
IMPULSE RESPONSE
OF A CIRCUIT WITH K=5

FIGURE 18

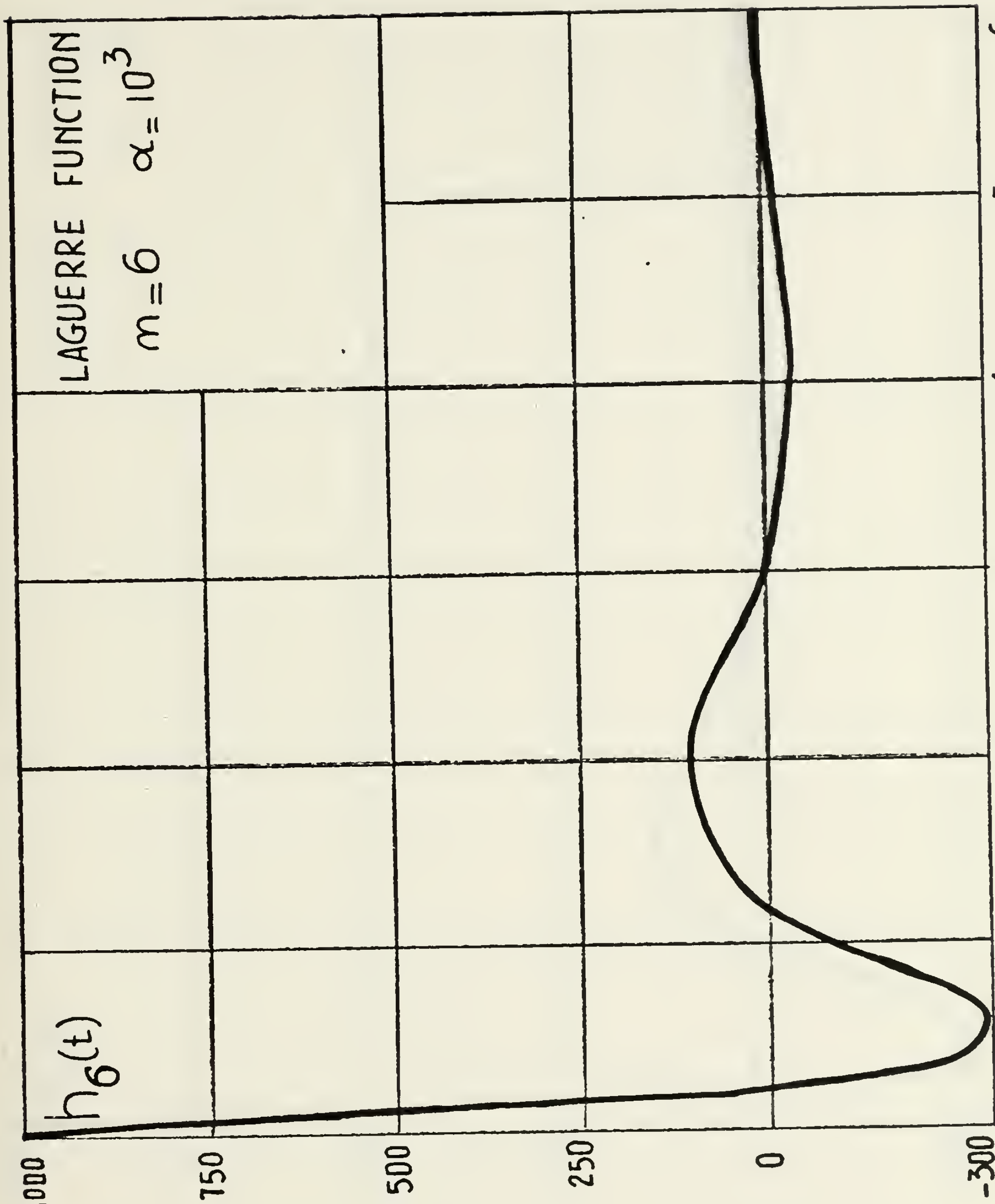

LAGUERRE FUNCTION
$m = 6$ $\alpha = 10^3$
$h_6(t)$
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 19

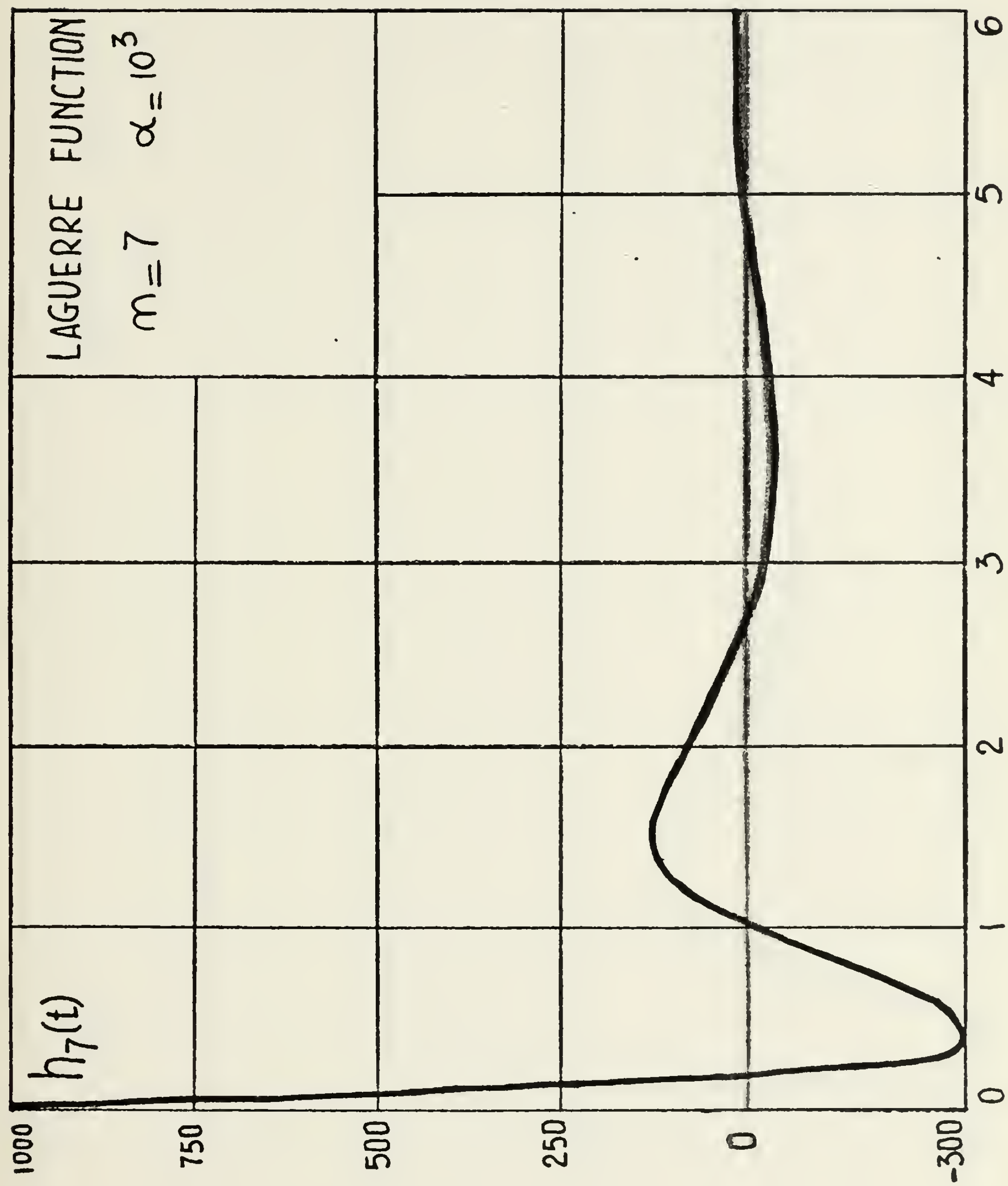

LAGUERRE FUNCTION
$m = 7$ $\alpha = 10^3$
$h_7(t)$
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 20

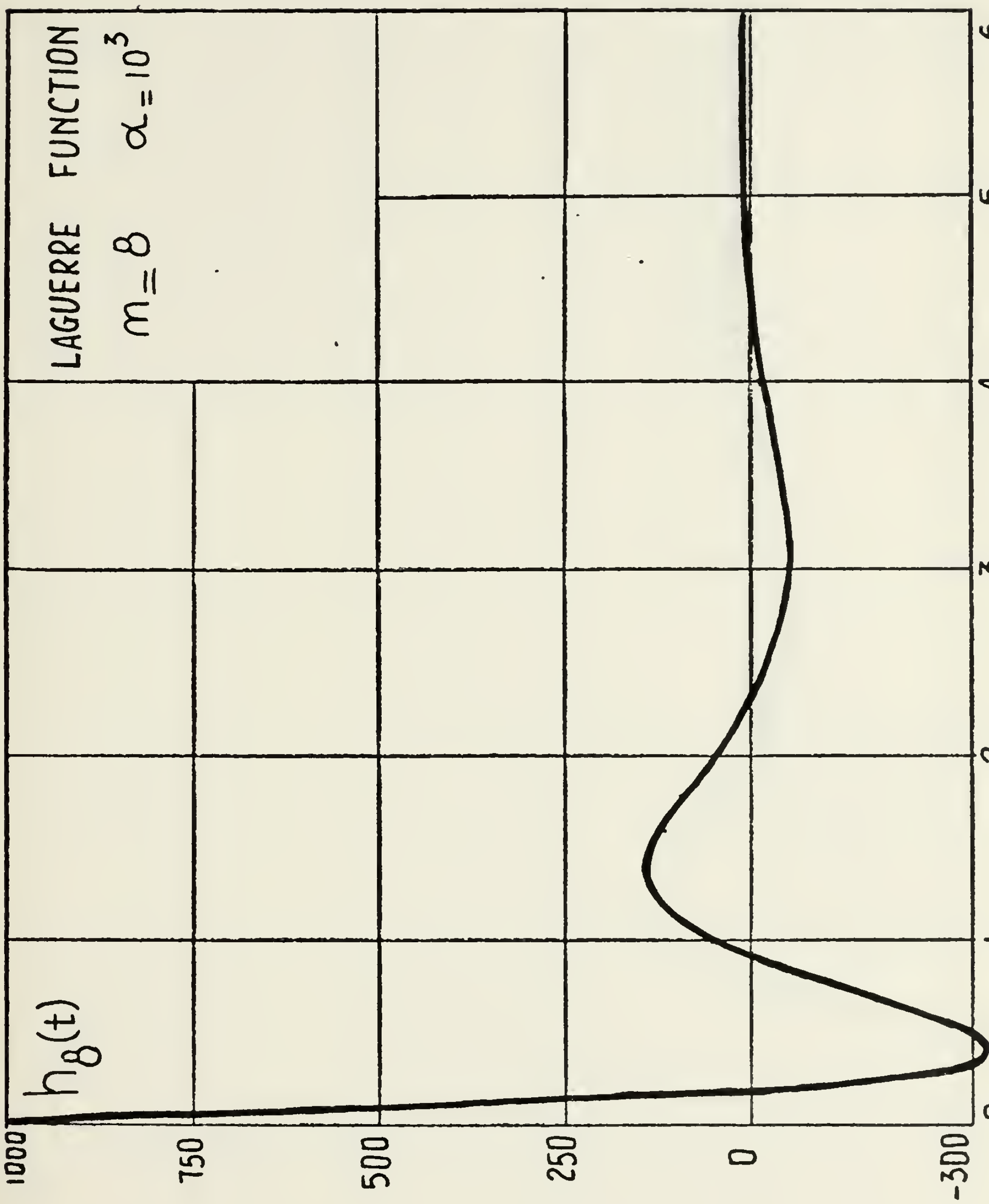

LAGUERRE FUNCTION
$m = 8$ $\alpha = 10^3$
$h_8(t)$
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 21

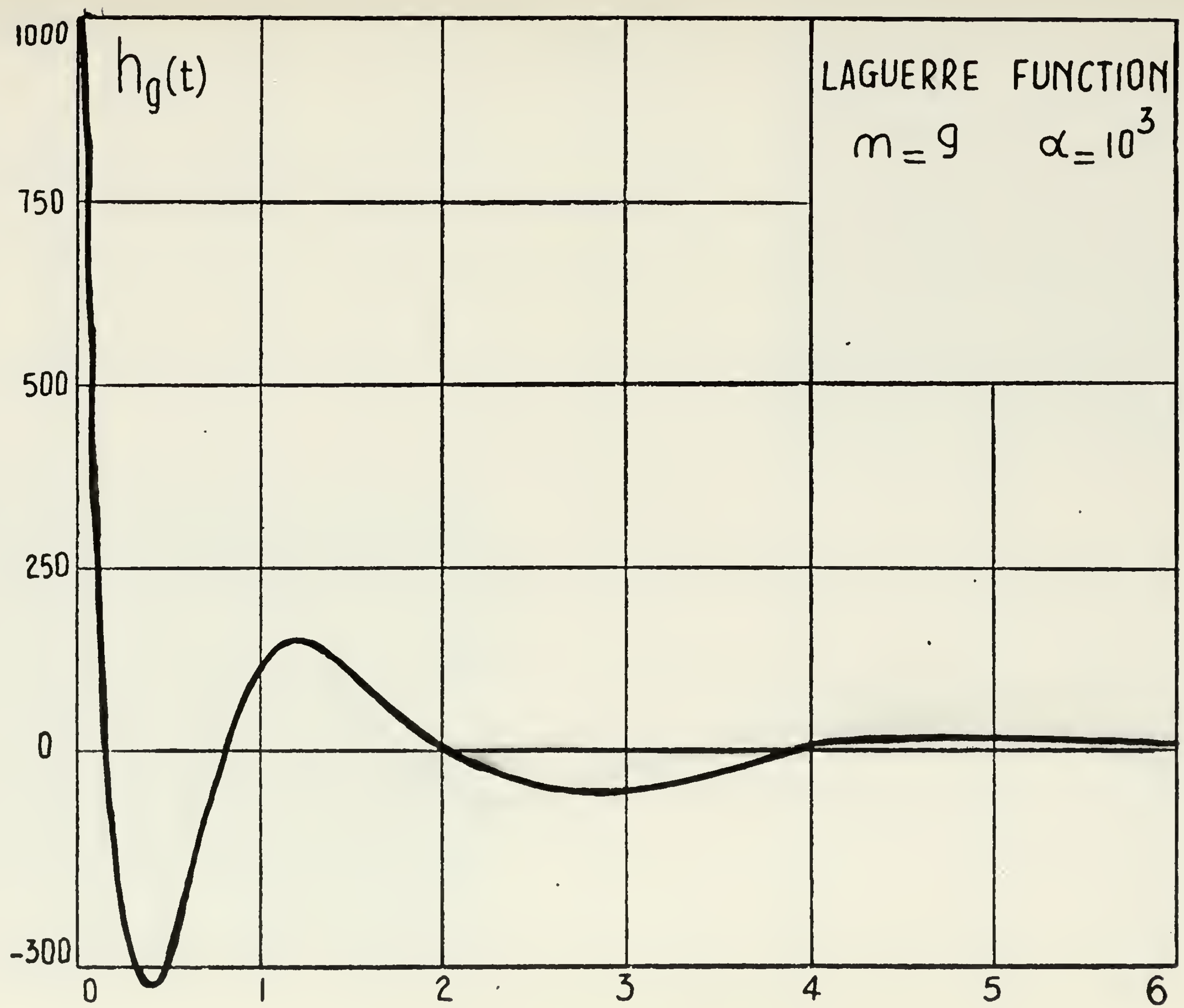
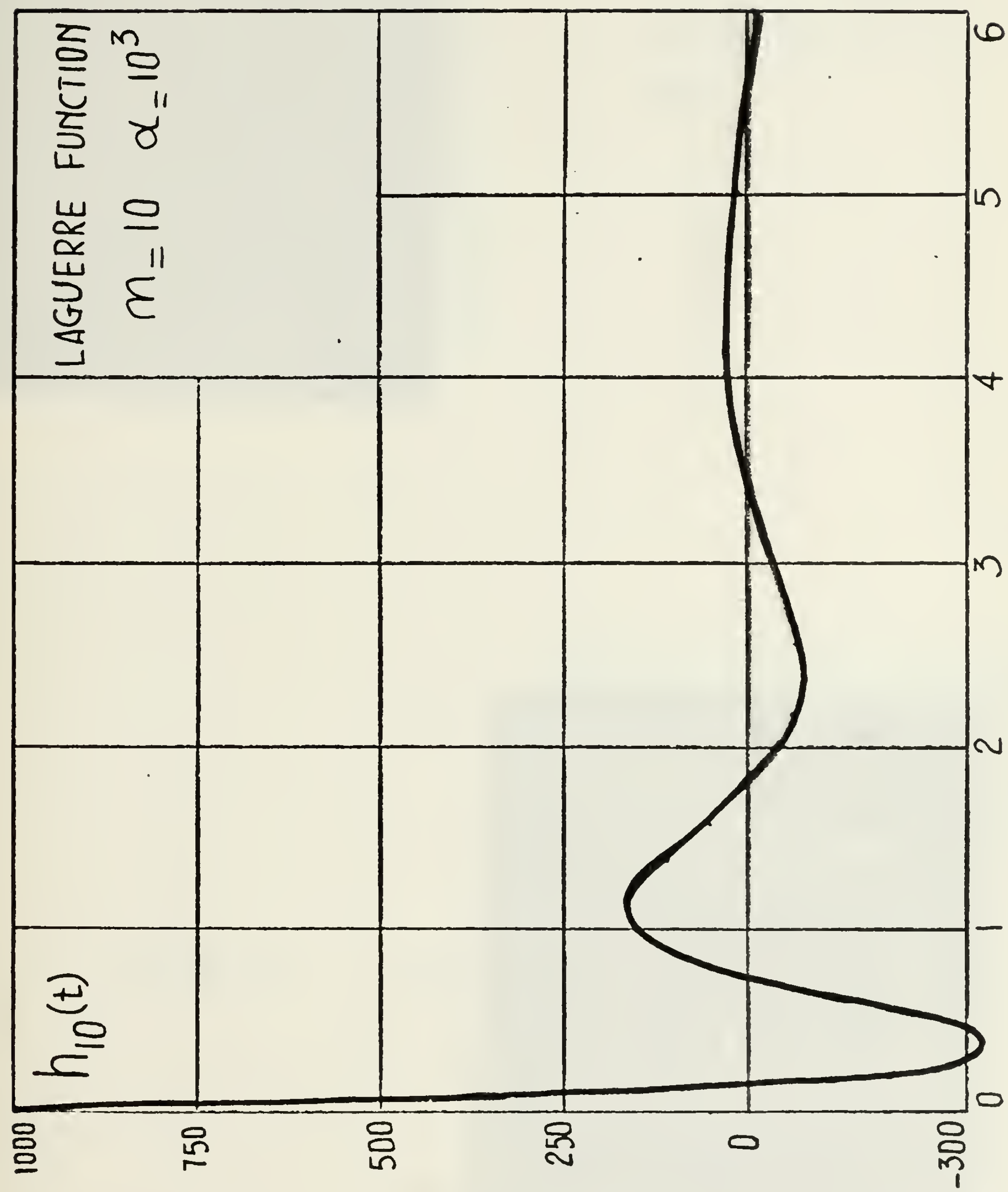
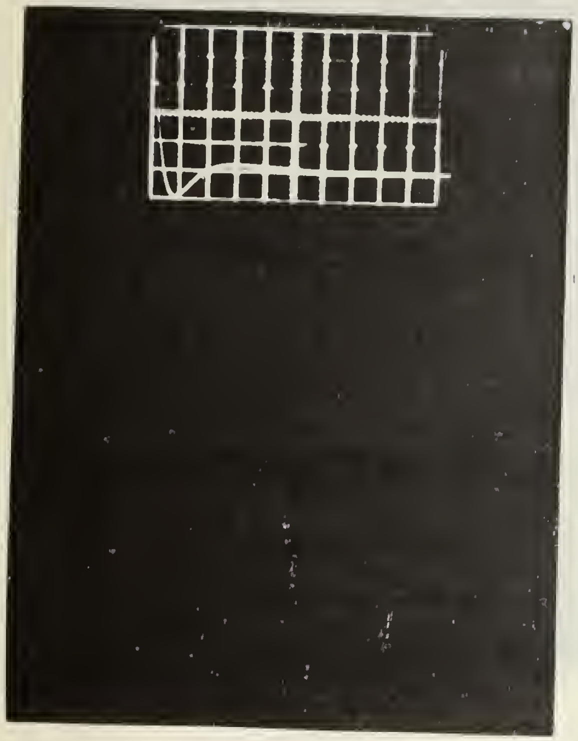

1000
750
500
250
0
-300
0
1
2
3
4
5
6
$h_g(t)$
LAGUERRE FUNCTION
$m = 9$ $\alpha = 10^3$

FIGURE 22



$h_{10}(t)$
LAGUERRE FUNCTION
$n = 10$ $\alpha = 10^3$
1000
750
500
250
0
-300
0
1
2
3
4
5
6

FIGURE 23



n=3

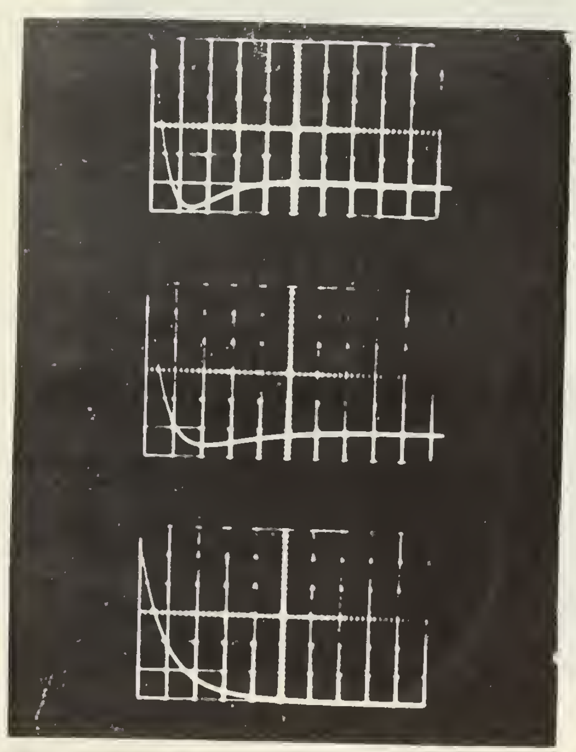
Impulse Responses
of Circuits with
k=10

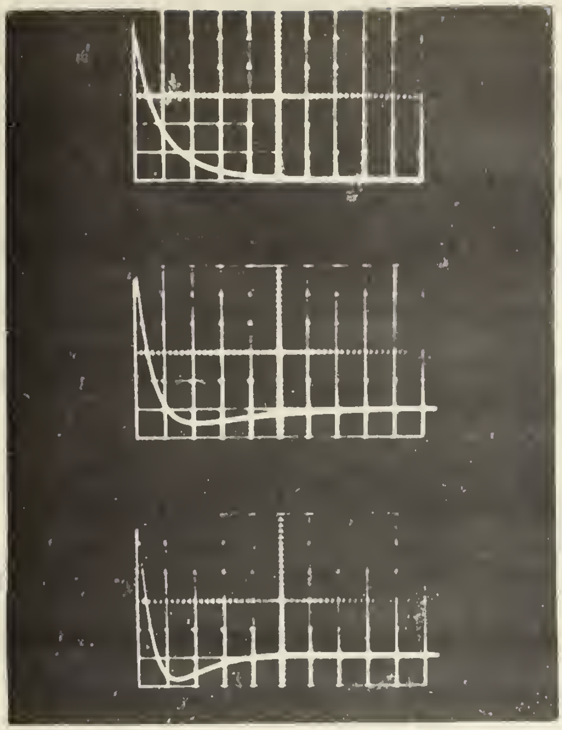
n=2

FIGURE 24

n=1

n=0



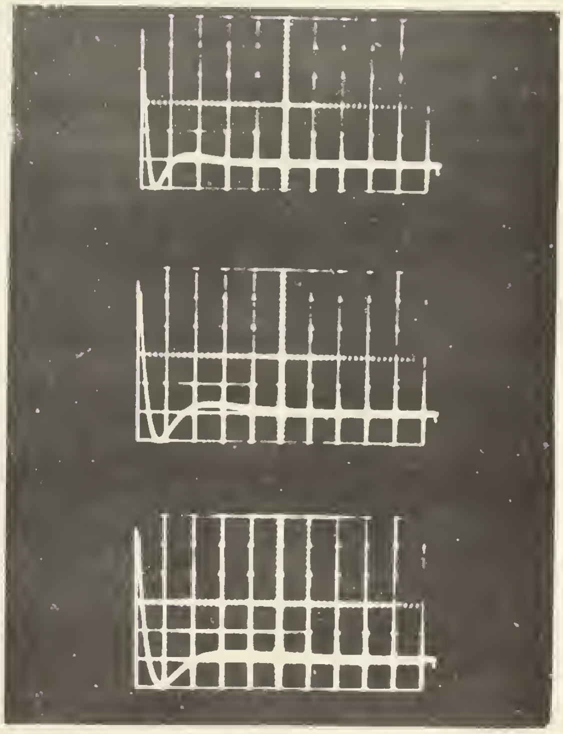


n=0

n=1

n=2

Impulse Responses of Circuits with k=5



n=5

n=4

n=3

FIGURE 25

VI. CONCLUSION

The realizability of the method of Laguerre type filters or without a "pure" time delay to obtain correlation functions was shown together with a set of very simple filters to achieve them for low frequency signals.

The following things require further investigation:

Multipliers and adder that could be used with the filters presented in this thesis to build a correlator as the one shown in Figure 5.

Factibility of building, through the techniques of Large Scale Integration, Laguerre type filters for several values of the base time constant alfa and with a behavior close to the ideal type of filter, which means that the circuits have to use buffer amplifiers.

APPENDIX

PROGRAM TO COMPUTE THE VALUES OF LAGUERRE FUNCTIONS

```
C
C                       MAIN PROGRAM
C
C     THIS PROGRAM CALCULATES THE LAGUERRE FUNCTIONS FOR
C     ORDERS 0 UP TO N AS A FUNCTION OF THE ARGUMENT X.
C
C     THE PRODUCT ALFA*TIME IS TAKEN AS THE ARGUMENT.
C
C     BASIC TIME CONSTANT:KALFA
C
C     NUMBER OF DISCRETE VALUES OF THE ARGUMENT:M
C
C     INCREMENT OF THE ARGUMENT:DELTA
C
      DIMENSION Y(350)
      READ(5,100)N
      READ(5,100)M
      READ(5,101)DELTA
C
C     STARTING VALUE OF ARGUMENT
C
      READ(5,101)X
      READ(5,102)KALFA
      L=N+1
      DO 30 I=1,M
      CALL LAP(Y,X,N)
      DAMP=KALFA*EXP(-X)
      DO 20 J=1,L
      Y(J)=Y(J)*DAMP
      WRITE(6,200)Y(J)
   20 CONTINUE
      X=X+DELTA
   30 CONTINUE
C
  100 FORMAT(I2)
  101 FORMAT(F5.3)
  102 FORMAT(I7)
  200 FORMAT(F9.2)
C
      STOP
      END
C
```

```
C
C                            SUBROUTINE
C
C       SUBROUTINE LAP
C
C       PURPOSE
C          COMPUTE THE VALUES OF THE LAGUERRE POLYNOMIALS
C          L(N,X) FOR ARGUMENT VALUE X AND ORDERS 0 UP TO
C          N
C
C       USAGE
C          CALL LAP(Y,X,N)
C
C       DESCRIPTION OF PARAMETERS
C          Y    -RESULT VECTOR OF DIMENSION N+1 CONTAINING
C                THE VALUES OF LAGUERRE POLYNOMIALS OF
C                ORDER 0 UP TO N FOR GIVEN ARGUMENT X
C                VALUES ARE ORDERED FROM LOW TO HIGH ORDER
C          X    -ARGUMENT OF LAGUERRE POLYNOMIAL
C          N    -ORDER OF LAGUERRE POLYNOMIAL
C
C
      SUBROUTINE LAP(Y,X,N)
C
      DIMENSION Y(1)
C
C        TEST OF ORDER
      Y(1)=1.
      IF(N)1,1,2
    1 RETURN
C
    2 Y(2)=1.-X
      IF(N-1)1,1,3
C
C        INITIALIZATION
    3 T=1.+X
C
      DO 4 I=2,N
    4 Y(I+1)=Y(I)-Y(I-1)+Y(I)-(T*Y(I)-Y(I-1))/FLOAT(I)
      RETURN
      END
```

LIST OF REFERENCES

1. Chan, S. P., Chan, S. Y., and Chan, S. G., Analysis of Linear Networks and Systems, p. 420-469, Addison-Wesley, 1972.

2. Lampard, D. G., "A New Method of Determining Correlation Functions of Stationary Time Series," Institution Monograph, p. 35-41, August 1954.

3. Lu Bow, B., "Correlation Entering New Fields with Real Time Signal Analysis," Electronics, p. 75-81, October 31, 1966.

4. Starkweather, D. W., "Linear Digital Correlator," Navy Electronics Laboratory, June 10, 1969.

5. Thomas, J. B., Statistical Communication Theory, p. 86-90, Wiley, 1969.

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This work deals with the correlation functions and their methods of obtention. A method without a "pure" time delay is considered which uses Laguerre functions type filters. The possibilities of utilization of the nonsymmetric Laguerre type filter are analyzed. A very inexpensive set of circuits which are useful to obtain the mentioned filters for low frequency signals are presented, together with their computer analysis and laboratory realizations.

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